Mathematics

For Class 7

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1 Integers

In class VI, we read about integers and various operations on them. We shall review them here and study the various properties satisfied by various operations on them.

Various Types of Numbers

Natural numbers *Counting numbers are called natural numbers.*
Thus, 1, 2, 3, 4, 5, 6, ..., etc., are all natural numbers.

Whole numbers *All natural numbers together with 0 (zero) are called whole numbers.*
Thus, 0, 1, 2, 3, 4, ..., etc., are whole numbers.
Clearly, every natural number is a whole number but 0 is a whole number which is not a natural number.

Integers *All natural numbers, 0 and negatives of counting numbers are called integers.*
Thus, ..., −4, −3, −2, −1, 0, 1, 2, 3, 4, ..., etc., are all integers.
(i) **Positive integers:** 1, 2, 3, 4, 5, ..., etc., are all positive integers.
(ii) **Negative integers:** −1, −2, −3, −4, ..., etc., are all negative integers.
(iii) **Zero** is an integer which is neither positive nor negative.

ADDITION OF INTEGERS

Rule 1. *If two positive or two negative integers are added, we add their values regardless of their signs and give the sum their common sign.*

EXAMPLE 1. *Add: (i) 36 and 27 (ii) −31 and −25*

Solution We have:

(i)
$$\begin{array}{r} +36 \\ +27 \\ \hline 63 \\ \hline \end{array}$$

(ii)
$$\begin{array}{r} -31 \\ -25 \\ \hline -56 \\ \hline \end{array}$$

∴ (i) $36 + 27 = 63$.
(ii) $(-31) + (-25) = -56$.

Rule 2. *To add a positive and a negative integer, we find the difference between their numerical values regardless of their signs and give the sign of the integer with the greater value to it.*

REMARK In order to add two integers of unlike signs, we see which is more and by how much.

EXAMPLE 2. *Add: (i) $-47 + 18$ (ii) $(-29) + 52$*

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Solution Using the rule for addition of integers with unlike signs, we have:

(i) -47
$+18$
-29

$\therefore\ (-47)+18=-29.$

(ii) -29
$+52$
$+23$

$\therefore\ (-29)+52=+23.$

PROPERTIES OF ADDITION OF INTEGERS

I. Closure Property of Addition: *The sum of two integers is always an integer.*

EXAMPLES (i) $5+4=9$, which is an integer.
(ii) $4+(-8)=-4$, which is an integer.
(iii) $(-3)+(-8)=-11$, which is an integer.
(iv) $15+(-9)=6$, which is an integer.
Hence, the sum of two integers is always an integer.

II. Commutative Law of Addition: If a and b are any two integers, then
$$a+b=b+a.$$

EXAMPLES (i) $(-4)+9=5$ and $9+(-4)=5$.
$\therefore\ (-4)+9=9+(-4)$.
(ii) $(-5)+(-8)=-13$ and $(-8)+(-5)=-13$.
$\therefore\ (-5)+(-8)=(-8)+(-5)$.

III. Associative Law of Addition: If a, b, c are any three integers, then
$$(a+b)+c=a+(b+c).$$

EXAMPLE Consider the integers (-6), (-8) and 5. We have:
$\{(-6)+(-8)\}+5=(-14)+5=-9.$
And, $(-6)+\{(-8)+5\}=(-6)+(-3)=-9.$
$\therefore\ \{(-6)+(-8)\}+5=(-6)+\{(-8)+5\}.$
Similarly, other examples may be taken up.

IV. Existence of Additive Identity: For any integer a, we have:
$$a+0=0+a=a.$$
0 is called the *additive identity* for integers.

EXAMPLES (i) $9+0=0+9=9$. (ii) $(-6)+0=0+(-6)=(-6)$.

V. Existence of Additive Inverse: For any integer a, we have:
$$a+(-a)=(-a)+a=0.$$
The *opposite* of an integer a is $(-a)$.
The sum of an integer and its opposite is 0.
Additive inverse of a is $(-a)$.
Similarly, *additive inverse of $(-a)$ is a.*

EXAMPLE We have: $5+(-5)=(-5)+5=0$.
So, the additive inverse of 5 is (-5).
And, the additive inverse of (-5) is 5.

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SUBTRACTION OF INTEGERS

For any integers a and b, we define:

(i) $a - b = a +$ (additive inverse of b) $= a + (-b)$.

(ii) $a - (-b) = a +$ [additive inverse of $(-b)$] $= a + b$.

SUMMARY

(i) $a - b = a + (-b)$

(ii) $a - (-b) = a + b$

EXAMPLE 1. *Subtract:*

(i) 9 *from* 4 *(ii)* -8 *from* 5 *(iii)* 7 *from* (-6) *(iv)* -9 *from* -5

Solution We have:

(i) $4 - 9 = 4 + (-9) = -5$.

(ii) $5 - (-8) = 5 + 8 = 13$.

(iii) $(-6) - 7 = (-6) + (-7) = -13$.

(iv) $-5 - (-9) = (-5) + 9 = 4$.

EXAMPLE 2. *Write:*

(i) a pair of integers whose sum is -8*;*

(ii) a pair of integers whose difference is -12*;*

(iii) a pair of integers whose sum is 0*;*

(iv) a negative integer and a positive integer whose sum is -6*;*

(v) a negative integer and a positive integer whose difference is -4.

Solution Clearly, we have:

(i) $(-3) + (-5) = -8$.

(ii) $(-15) - (-3) = (-15) + 3 = -12$.

(iii) $8 + (-8) = 0$.

(iv) $(-8) + 2 = (-6)$.

(v) $(-1) - 3 = -4$.

PROPERTIES OF SUBTRACTION OF INTEGERS

I. Closure Property for Subtraction: *If a and b are any two integers, then* $(a - b)$ *is always an integer.*

EXAMPLES (i) $2 - 5 = 2 + (-5) = -3$, which is an integer.

(ii) $(-2) - 6 = (-2) + (-6) = -8$, which is an integer.

(iii) $3 - (-5) = 3 + 5 = 8$, which is an integer.

(iv) $-4 - (-6) = -4 + 6 = 2$, which is an integer.

II. Subtraction of Integers is Not Commutative.

EXAMPLES (i) Consider the integers 3 and 5. We have:

$(3 - 5) = 3 + (-5) = -2$ and $(5 - 3) = 5 + (-3) = 2$.

$\therefore \quad (3 - 5) \neq (5 - 3)$.

(ii) Consider the integers (-4) and 2. We have:

$(-4) - 2 = (-4) + (-2) = -6$ and $2 - (-4) = (2 + 4) = 6$.

$\therefore \quad (-4) - 2 \neq 2 - (-4)$.

(iii) Consider the integers (-6) and (-4). We have:

$(-6) - (-4) = (-6) + 4 = -2$ and $(-4) - (-6) = (-4) + 6 = 2$.

$\therefore \quad (-6) - (-4) \neq (-4) - (-6)$.

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III. Subtraction of Integers is Not Associative.

EXAMPLE Consider the integers 3, (−4) and (−5). We have:

$\{3-(-4)\}-(-5)=(3+4)-(-5)=7-(-5)=(7+5)=12.$

And, $3-\{(-4)-(-5)\}=3-\{(-4)+5\}=(3-1)=2.$

$\therefore \quad \{3-(-4)\}-(-5)\neq 3-\{(-4)-(-5)\}.$

EXAMPLE 3. *Write a pair of integers whose difference gives*

(i) zero;

(ii) a negative integer;

(iii) an integer smaller than both the integers;

(iv) an integer greater than both the integers;

(v) an integer greater than only one of the integers.

Solution (i) Consider the integers 5 and 5.

Clearly, $(5-5)=0$.

(ii) Consider the integers 4 and 6. Then,

$(4-6)=4+(-6)=-2$, which is a negative integer.

(iii) Consider the integers (−6) and 4. Then,

$(-6)-4=(-6)+(-4)=-10.$

Thus, we get an integer smaller than both the integers.

(iv) Consider the integers 5 and (−3). Then,

$5-(-3)=(5+3)=8.$

Thus, we get an integer greater than both the integers.

(v) Consider the integers (−5) and (−2). Then,

$(-5)-(-2)=(-5)+2=-3.$

Clearly, $-3>-5$ but -3 is not greater than -2.

EXAMPLE 4. *The sum of two integers is −11. If one of them is 9, find the other.*

Solution Let the other integer be a. Then,

$9+a=-11 \Rightarrow a=(-11)-9=(-11)+(-9)=-20.$

Hence, the other integer is −20.

EXAMPLE 5. *The difference of an integer a and (−5) is −3. Find the value of a.*

Solution We have:

$a-(-5)=-3 \Rightarrow a+5=(-3)$

$\Rightarrow a=(-3)-5=(-3)+(-5)=-8.$

Hence, $a=-8$.

EXERCISE 1A

1. Evaluate:

(i) $15+(-8)$ (ii) $(-16)+9$ (iii) $(-7)+(-23)$

(iv) $(-32)+47$ (v) $53+(-26)$ (vi) $(-48)+(-36)$

2. Find the sum of:

(i) 153 and −302 (ii) 1005 and −277 (iii) −2035 and 297

(iv) −489 and −324 (v) −1000 and 438 (vi) −238 and 500

3. Find the additive inverse of:

(i) −83 (ii) 256 (iii) 0 (iv) −2001

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4. Subtract:

(i) 28 from −42 (ii) −36 from 42 (iii) −37 from −53

(iv) −66 from −34 (v) 318 from 0 (vi) −153 from −240

(vii) −64 from 0 (viii) −56 from 144

5. Subtract the sum of −1032 and 878 from −34.

6. Subtract −134 from the sum of 38 and −87.

7. Fill in the blanks:

(i) $\{(-13)+27\}+(-41)=(-13)+\{27+(\ldots\ldots)\}$

(ii) $(-26)+\{(-49)+(-83)\}=\{(-26)+(-49)\}+(\ldots\ldots)$

(iii) $53+(-37)=(-37)+(\ldots\ldots)$

(iv) $(-68)+(-76)=(\ldots\ldots)+(-68)$

(v) $(-72)+(\ldots\ldots)=-72$

(vi) $-(-83)=\ldots\ldots$

(vii) $(-60)-(\ldots\ldots)=-59$

(viii) $(-31)+(\ldots\ldots)=-40$

8. Simplify: $\{-13-(-27)\}+\{-25-(-40)\}$.

9. Find $36-(-64)$ and $(-64)-36$. Are they equal?

10. If $a=-8$, $b=-7$, $c=6$, verify that $(a+b)+c=a+(b+c)$.

11. If $a=-9$ and $b=-6$, show that $(a-b)\neq(b-a)$.

12. The sum of two integers is −16. If one of them is 53, find the other.

13. The sum of two integers is 65. If one of them is −31, find the other.

14. The difference of an integer a and (−6) is 4. Find the value of a.

15. Write a pair of integers whose sum gives

(i) zero;

(ii) a negative integer;

(iii) an integer smaller than both the integers;

(iv) an integer greater than both the integers;

(v) an integer smaller than only one of the integers.

Hint. *(i)* 6 *and* (−6) *(ii)* 4 *and* (−9) *(iii)* (−3) *and* (−5) *(iv)* 4 *and* 5 *(v)* 5 *and* (−3)

16. For each of the following statements, write (T) for true and (F) for false:

(i) The smallest integer is zero.

(ii) −10 is greater than −7.

(iii) Zero is larger than every negative integer.

(iv) The sum of two negative integers is a negative integer.

(v) The sum of a negative integer and a positive integer is always a positive integer.

MULTIPLICATION OF INTEGERS

Rule 1. *To find the product of two integers with unlike signs, we find the product of their values regardless of their signs and give a minus sign to the product.*

EXAMPLE 1. *Find each of the following products:*

(i) $6\times(-5)$ *(ii)* $(-7)\times9$ *(iii)* $35\times(-18)$ *(iv)* $(-42)\times20$

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Solution We have:

(i) $6 \times (-5) = -30$.

(ii) $(-7) \times 9 = -63$.

(iii) $35 \times (-18) = -(35 \times 18) = -630$.

(iv) $(-42) \times 20 = -(42 \times 20) = -840$.

Rule 2. *To find the product of two integers with the same sign, we find the product of their values regardless of their signs and give a plus sign to the product.*

EXAMPLE 2. *Find each of the following products:*

(i) 12×16 *(ii)* $(-8) \times (-14)$ *(iii)* $(-25) \times (-19)$ *(iv)* $(-70) \times (-31)$

Solution We have:

(i) $(12 \times 16) = 192$.

(ii) $(-8) \times (-14) = (8 \times 14) = 112$.

(iii) $(-25) \times (-19) = (25 \times 19) = 475$.

(iv) $(-70) \times (-31) = (70 \times 31) = 2170$.

PROPERTIES OF MULTIPLICATION OF INTEGERS

I. Closure Property for Multiplication: *The product of two integers is always an integer.*

EXAMPLES (i) $7 \times 5 = 35$, which is an integer.

(ii) $(-8) \times 4 = -32$, which is an integer.

(iii) $9 \times (-6) = -54$, which is an integer.

(iv) $(-8) \times (-7) = 56$, which is an integer.

II. Commutative Law for Multiplication: For any two integers a and b, we have:

$$(a \times b) = (b \times a).$$

EXAMPLES (i) $5 \times (-8) = -40$ and $(-8) \times 5 = -40$.

$\therefore \quad 5 \times (-8) = (-8) \times 5$.

(ii) $(-9) \times (-7) = 63$ and $(-7) \times (-9) = 63$.

$\therefore \quad (-9) \times (-7) = (-7) \times (-9)$.

III. Associative Law for Multiplication: For any integers a, b, c, we have:

$$(a \times b) \times c = a \times (b \times c).$$

EXAMPLES (i) Consider the integers 3, –5 and –8. We have:

$\{3 \times (-5)\} \times (-8) = (-15) \times (-8) = 120$

and $3 \times \{(-5) \times (-8)\} = (3 \times 40) = 120$.

$\therefore \quad \{3 \times (-5)\} \times (-8) = 3 \times \{(-5) \times (-8)\}$.

(ii) Consider the integers (-8), (-6) and (-5). We have:

$\{(-8) \times (-6)\} \times (-5) = 48 \times (-5) = -240$

and $(-8) \times \{(-6) \times (-5)\} = (-8) \times 30 = -240$.

$\therefore \quad \{(-8) \times (-6)\} \times (-5) = (-8) \times \{(-6) \times (-5)\}$.

IV. Distributive Law of Multiplication over Addition: For any integers a, b, c, we have:

$$a \times (b + c) = (a \times b) + (a \times c).$$

EXAMPLES (i) Consider the integers 5, (–6) and (–8). We have:

$5 \times \{(-6) + (-8)\} = 5 \times (-14) = -70$

and $\{5 \times (-6)\} + \{5 \times (-8)\} = (-30) + (-40) = -70$.

$\therefore \quad 5 \times \{(-6) + (-8)\} = \{5 \times (-6)\} + \{5 \times (-8)\}$.

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(ii) Consider the integers (−5), (−7) and (−9). We have:

$(-5)\times\{(-7)+(-9)\}=(-5)\times(-16)=80$

and $\{(-5)\times(-7)\}+\{(-5)\times(-9)\}=(35+45)=80$.

$\therefore\quad (-5)\times\{(-7)+(-9)\}=\{(-5)\times(-7)\}+\{(-5)\times(-9)\}$.

V. Existence of Multiplicative Identity: For every integer a, we have: $(a\times1)=(1\times a)=a$.

1 is called the *multiplicative identity* for integers.

EXAMPLES (i) $(12\times1)=12$. (ii) $(-16)\times1=-16$.

VI. Existence of Multiplicative Inverse: Multiplicative inverse of a nonzero integer a is the number $\frac{1}{a}$, as $a\cdot\left(\frac{1}{a}\right)=\left(\frac{1}{a}\right)\cdot a=1$.

EXAMPLES (i) Multiplicative inverse of 6 is $\frac{1}{6}$.

(ii) Multiplicative inverse of −6 is $-\frac{1}{6}$.

VII. Property of Zero: For every integer a, we have :

$$(a\times0)=(0\times a)=0.$$

EXAMPLES (i) $8\times0=0\times8=0$. (ii) $(-6)\times0=0\times(-6)=0$.

EXAMPLE 3. *Simplify:*

(i) $8\times(-15)+8\times6$ *(ii)* $(-18)\times7+(-18)\times(-4)$

(iii) $15\times(-32)+15\times(-18)$ *(iv)* $16\times(-9)+(-8)\times(-9)$

Solution Using the distributive laws, we get:

(i) $8\times(-15)+8\times6=8\times\{(-15)+6\}$ [$\because a\times b+a\times c=a\times(b+c)$]

$=8\times(-9)$

$=-72$.

(ii) $(-18)\times7+(-18)\times(-4)=(-18)\times\{7+(-4)\}$ [$\because a\times b+a\times c=a\times(b+c)$]

$=(-18)\times3$

$=-54$.

(iii) $15\times(-32)+15\times(-18)=15\times\{(-32)+(-18)\}$ [$\because a\times b+a\times c=a\times(b+c)$]

$=15\times(-50)$

$=-750$.

(iv) $16\times(-9)+(-8)\times(-9)=\{16+(-8)\}\times(-9)$ [$\because a\times c+b\times c=(a+b)\times c$]

$=8\times(-9)$

$=-72$.

IMPORTANT RESULTS

(i) $(-a_1)\times(-a_2)\times(-a_3)\times\ldots\times(-a_n)=-(a_1\times a_2\times a_3\times\ldots\times a_n)$**, when n is odd.**

(ii) $(-a_1)\times(-a_2)\times(-a_3)\times\ldots(-a_n)=(a_1\times a_2\times a_3\times\ldots\times a_n)$**, when n is even.**

(iii) $(-a)\times(-a)\times(-a)\times\ldots$ **n times** $=-a^n$**, when n is odd.**

(iv) $(-a)\times(-a)\times(-a)\times\ldots$ **n times** $=a^n$**, when n is even.**

(v) $(-1)\times(-1)\times(-1)\times\ldots$ **n times = −1, when n is odd.**

(vi) $(-1)\times(-1)\times(-1)\times\ldots$ **n times = 1, when n is even.**

EXAMPLE 4. *Evaluate:*

(i) $(-1)\times(-2)\times(-3)\times(-4)\times(-5)$ *(ii)* $(-3)\times(-5)\times(-2)\times(-4)$

(iii) $(-2)\times(-2)\times(-2)\times\ldots$ 8 *times* *(iv)* $(-2)\times(-2)\times(-2)\times\ldots$ 9 *times*

(v) $(-1)\times(-1)\times(-1)\times\ldots$ 100 *times* *(vi)* $(-1)\times(-1)\times(-1)\times\ldots$ 301 *times*

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Solution (i) Number of negative integers in the given product is odd.
Therefore, their product is negative.
$\therefore \quad (-1)\times(-2)\times(-3)\times(-4)\times(-5) = -(1\times2\times3\times4\times5) = -120.$

(ii) Number of negative integers in the given product is even.
Therefore, their product is positive.
$\therefore \quad (-3)\times(-5)\times(-2)\times(-4) = (3\times5\times2\times4) = 120.$

(iii) Number of negative integers in the given product is even.
Therefore, their product is positive.
$\therefore \quad (-2)\times(-2)\times(-2)\times\ldots 8 \text{ times} = 2^8 = 256.$

(iv) Number of negative integers in the given product is odd.
Therefore, their product is negative.
$\therefore \quad (-2)\times(-2)\times(-2)\times\ldots 9 \text{ times} = -2^9 = -512.$

(v) Number of negative integers in the given product is even.
Therefore, their product is positive.
$\therefore \quad (-1)\times(-1)\times(-1)\times\ldots 100 \text{ times} = 1.$

(vi) Number of negative integers in the given product is odd.
Therefore, their product is negative.
$\therefore \quad (-1)\times(-1)\times(-1)\times\ldots 301 \text{ times} = -1.$

EXAMPLE 5. *What will be the sign of the product if we multiply together* 199 *negative and* 10 *positive integers?*

Solution Whatever may be the number of positive integers, it will not affect the sign of the product.

Since 199 is odd and the product of odd number of negative integers is negative, so the given product is negative.

EXAMPLE 6. *In a class test containing* 20 *questions,* 4 *marks are given for every correct answer and* (–2) *marks are given for every incorrect answer. Ranjita attempts all questions and* 12 *of her answers are correct. What is her total score?*

Solution Marks given for 1 correct answer = 4.

Marks given for 12 correct answers $= (4\times12) = 48.$

Marks given for 1 incorrect answer = –2.

Marks given for $(20-12)$, i.e., 8 incorrect answers $= (-2)\times8 = -16.$

Ranjita's total score $= 48 + (-16) = 32.$

EXAMPLE 7. *A shopkeeper gains* ₹ 1 *on each pen and loses* 40 *paise on each pencil. He sells* 45 *pens and some pencils losing* ₹ 5 *in all. How many pencils does he sell?*

Solution Suppose he sells x pencils.

Total gain on pens = ₹ 45.

Total loss on pencils $= ₹\,\frac{40x}{100} = ₹\,\frac{2x}{5}.$

$$\therefore \quad 45 - \frac{2x}{5} = -5 \Rightarrow \frac{2x}{5} = (45+5)$$

$$\Rightarrow \frac{2x}{5} = 50 \Rightarrow x = \frac{(50\times5)}{2} = 125.$$

Hence, the number of pencils sold is 125.

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EXAMPLE 8. *A certain freezing process requires that room temperature be lowered from 40°C at the rate of 5°C per hour. What will be the room temperature 12 hours after the process begins?*

Solution Temperature after n hours = $(40-5n)$°C.

∴ temperature after 12 hours = $(40-5\times12)$°C

$= (40-60)$°C = -20°C.

Hence, the room temperature after 12 hours would be -20°C.

EXERCISE 1B

1. Multiply:

(i) 16 by 9 (ii) 18 by -6 (iii) 36 by -11 (iv) -28 by 14

(v) -53 by 18 (vi) -35 by 0 (vii) 0 by -23 (viii) -16 by -12

(ix) -105 by -8 (x) -36 by -50 (xi) -28 by -1 (xii) 25 by -11

2. Find each of the following products:

(i) $3\times4\times(-5)$ (ii) $2\times(-5)\times(-6)$ (iii) $(-5)\times(-8)\times(-3)$

(iv) $(-6)\times6\times(-10)$ (v) $7\times(-8)\times3$ (vi) $(-7)\times(-3)\times4$

3. Find each of the following products:

(i) $(-4)\times(-5)\times(-8)\times(-10)$ (ii) $(-6)\times(-5)\times(-7)\times(-2)\times(-3)$

(iii) $(-60)\times(-10)\times(-5)\times(-1)$ (iv) $(-30)\times(-20)\times(-5)$

(v) $(-3)\times(-3)\times(-3)\times\ldots$ 6 times (vi) $(-5)\times(-5)\times(-5)\times\ldots$ 5 times

(vii) $(-1)\times(-1)\times(-1)\times\ldots$ 200 times (viii) $(-1)\times(-1)\times(-1)\times\ldots$ 171 times

4. What will be the sign of the product, if we multiply 90 negative integers and 9 positive integers?

5. What will be the sign of the product, if we multiply 103 negative integers and 65 positive integers?

6. Simplify:

(i) $(-8)\times9+(-8)\times7$ (ii) $9\times(-13)+9\times(-7)$

(iii) $20\times(-16)+20\times14$ (iv) $(-16)\times(-15)+(-16)\times(-5)$

(v) $(-11)\times(-15)+(-11)\times(-25)$ (vi) $10\times(-12)+5\times(-12)$

(vii) $(-16)\times(-8)+(-4)\times(-8)$ (viii) $(-26)\times72+(-26)\times28$

7. Fill in the blanks:

(i) $(-6)\times(\ldots\ldots)=6$ (ii) $(-18)\times(\ldots\ldots)=(-18)$

(iii) $(-8)\times(-9)=(-9)\times(\ldots\ldots)$ (iv) $7\times(-3)=(-3)\times(\ldots\ldots)$

(v) $\{(-5)\times3\}\times(-6)=(\ldots\ldots)\times\{3\times(-6)\}$ (vi) $(-5)\times(\ldots\ldots)=0$

8. In a class test containing 10 questions, 5 marks are awarded for every correct answer and (-2) marks are awarded for every incorrect answer and 0 for each question not attempted.

(i) Ravi gets 4 correct and 6 incorrect answers. What is his score?

(ii) Reenu gets 5 correct and 5 incorrect answers. What is her score?

(iii) Heena gets 2 correct and 5 incorrect answers. What is her score?

9. Which of the following statements are true and which are false?

(i) The product of a positive and a negative integer is negative.

(ii) The product of two negative integers is a negative integer.

(iii) The product of three negative integers is a negative integer.

(iv) Every integer when multiplied with -1 gives its multiplicative inverse.

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(v) Multiplication on integers is commutative.
(vi) Multiplication on integers is associative.
(vii) Every nonzero integer has a multiplicative inverse as an integer.

DIVISION OF INTEGERS

We know that division is an inverse process of multiplication.

Rule 1. *For dividing one integer by the other, the two having unlike signs, we divide their values regardless of their signs and give a minus sign to the quotient.*

EXAMPLE 1. *Evaluate:*

(i) $(-48) \div 12$ *(ii)* $144 \div (-16)$ *(iii)* $(-69) \div 23$

Solution We have:

(i) $(-48) \div 12 = \frac{-48}{12} = -4.$

(ii) $144 \div (-16) = \frac{144}{(-16)} = -9.$

(iii) $(-69) \div 23 = \frac{(-69)}{23} = -3.$

Rule 2. *For dividing one integer by the other having like signs, we divide their values regardless of their signs and give a plus sign to the quotient.*

EXAMPLE 2. *Ealuate:*

(i) $98 \div 14$ *(ii)* $(-48) \div (-16)$ *(iii)* $(-90) \div (-15)$

Solution We have:

(i) $98 \div 14 = \frac{98}{14} = 7.$

(ii) $(-48) \div (-16) = \frac{-48}{-16} = 3.$

(iii) $(-90) \div (-15) = \frac{-90}{-15} = 6.$

EXAMPLE 3. *Evaluate:*

(i) $(-133) \div 19$ *(ii)* $168 \div (-14)$ *(iii)* $(-336) \div (-21)$

Solution We have:

(i) $(-133) \div 19 = \frac{-133}{19} = -7.$

(ii) $168 \div (-14) = \frac{168}{-14} = -12.$

(iii) $(-336) \div (-21) = \frac{-336}{-21} = 16.$

EXAMPLE 4. *Fill in the blanks:*

(i) $(-37) \div (\ldots\ldots) = 1$ *(ii)* $(\ldots\ldots) \div 36 = -2$

(iii) $(\ldots\ldots) \div 69 = 0$ *(iv)* $(\ldots\ldots) \div (-18) = -5$

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Solution (i) Clearly, $(-37) \div (-37) = 1$.

Hence, the required number is (-37).

(ii) Let the required number be x. Then,

$$x \div 36 = -2 \;\Rightarrow\; \frac{x}{36} = -2$$
$$\Rightarrow\; x = 36 \times (-2) = -72.$$

Hence, the required number is -72.

(iii) When 0 is divided by any nonzero number, then the quotient is 0 (see IV below).

$\therefore\;\; 0 \div 69 = 0.$

(iv) Let the required number be x. Then,

$$x \div (-18) = -5 \;\Rightarrow\; \frac{x}{-18} = -5$$
$$\Rightarrow\; x = (-18) \times (-5) = 90.$$

Hence, the required number is 90.

Modulus of An Integer

The modulus of an integer a, denoted by $|a|$ is defined as

$$|a| = \begin{cases} a, & \text{if } a \text{ is positive or zero.} \\ -a, & \text{if } a \text{ is negative.} \end{cases}$$

Thus, $|6| = 6$ and $|-6| = -(-6) = 6$.

Distance between the Two Points

Let A and B be two points at distances a and b respectively from the origin. Then, we define $AB = |a - b|$.

EXAMPLE 5. *An elevator descends into a mine shaft at the rate of 6 m/min. If the descent starts from 20 m above the ground level, how long will it take to reach −370 m ?*

Solution Let the point O denote the ground level.

Then, $OA = 20$ m and $OB = -370$ m.

$\therefore\;\; AB = |OA - OB| = |20 - (-370)| = |20 + 370| = 390$ m.

$\therefore\;\;$ distance covered = 390 m.

Rate of descent = 6 m/min.

Time taken $= \frac{390}{6}$ min = 65 min = 1 hr 5 min.

PROPERTIES OF DIVISION OF INTEGERS

I. *If a and b are integers then $(a \div b)$ is not necessarily an integer.*

EXAMPLES (i) 16 and 5 are both integers, but $(16 \div 5)$ is not an integer.

(ii) (-9) and 4 are both integers, but $\{(-9) \div 4\}$ is not an integer.

II. *If a is an integer and $a \neq 0$, then $a \div a = 1$.*

EXAMPLES (i) $16 \div 16 = 1$. (ii) $(-8) \div (-8) = 1$.

III. *If a is an integer, then $(a \div 1) = a$.*

EXAMPLES (i) $7 \div 1 = 7$. (ii) $(-6) \div 1 = (-6)$.

IV. *If a is an integer and $a \neq 0$, then $(0 \div a) = 0$ but $(a \div 0)$ is not meaningful.*

EXAMPLES (i) $0 \div 6 = 0$. (ii) $0 \div (-4) = 0$. (iii) $6 \div 0$ is meaningless.

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V. *If a, b, c are integers, then* $(a \div b) \div c \neq a \div (b \div c)$, *unless* $c = 1$.

Thus, division on integers is not associative.

EXAMPLE Let $a = -8$, $b = 4$ and $c = -2$. Then,

$$(a \div b) \div c = \{(-8) \div 4\} \div (-2) = (-2) \div (-2) = 1.$$
$$a \div (b \div c) = (-8) \div \{4 \div (-2)\} = (-8) \div (-2) = 4.$$
$$\therefore \quad (a \div b) \div c \neq a \div (b \div c).$$

However, if $a = -8$, $b = 4$ and $c = 1$, then

$$(a \div b) \div c = \{(-8) \div 4)\} \div 1 = (-2) \div 1 = (-2).$$
$$a \div (b \div c) = (-8) \div \{4 \div 1\} = (-8) \div 4 = (-2).$$

So, in this case, $(a \div b) \div c = a \div (b \div c)$.

VI. *If a, b, c are nonzero integers and* $a > b$, *then*

(i) $(a \div c) > (b \div c)$, if c is positive.

(ii) $(a \div c) < (b \div c)$, if c is negative.

EXAMPLES (i) $27 > 18$ and 9 is positive.

$$\therefore \quad \frac{27}{9} > \frac{18}{9}.$$

(ii) $27 > 18$ and (-9) is negative.

$$\therefore \quad \frac{27}{-9} < \frac{18}{-9}.$$

EXERCISE 1C

1. Divide:

(i) 65 by −13 (ii) −84 by 12 (iii) −76 by 19 (iv) −132 by 12
(v) −150 by 25 (vi) −72 by −18 (vii) −105 by −21 (viii) −36 by −1
(ix) 0 by −31 (x) −63 by 63 (xi) −23 by −23 (xii) −8 by 1

2. Fill in the blanks:

(i) $72 \div (......) = -4$ (ii) $-36 \div (......) = -4$ (iii) $(......) \div (-4) = 24$
(iv) $(......) \div 25 = 0$ (v) $(......) \div (-1) = 36$ (vi) $(......) \div 1 = -37$
(vii) $39 \div (......) = -1$ (viii) $1 \div (......) = -1$ (ix) $-1 \div (......) = -1$

3. Write (T) for true and (F) for false for each of the following statements:

(i) $0 \div (-4) = 0$ (ii) $(-6) \div 0 = 0$ (iii) $(-5) \div (-1) = -5$
(iv) $(-8) \div 1 = -8$ (v) $(-1) \div (-1) = -1$ (vi) $(-9) \div (-1) = 9$

EXERCISE 1D

OBJECTIVE QUESTIONS

Mark (✓) against the correct answer in each of the following:

1. $6 - (-8) = ?$

(a) −2 (b) 2 (c) 14 (d) none of these

2. $-9 - (-6) = ?$

(a) −15 (b) −3 (c) 3 (d) none of these

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3. By how much does 2 exceed –3?

(a) –1 (b) 1 (c) –5 (d) 5

Hint. *Required number* $= 2-(-3)$.

4. What must be subtracted from –1 to get –6?

(a) 5 (b) –5 (c) 7 (d) –7

Hint. $-1-x=-6 \Rightarrow x=-1+6$.

5. How much less than –2 is –6?

(a) 4 (b) –4 (c) 8 (d) –8

Hint. *Required number* $= (-2)-(-6)$.

6. On subtracting 4 from –4, we get

(a) 8 (b) –8 (c) 0 (d) none of these

7. By how much does –3 exceed –5?

(a) –2 (b) 2 (c) 8 (d) –8

Hint. *Required number* $= (-3)-(-5) = -3+5$.

8. What must be subtracted from –3 to get –9?

(a) –6 (b) 12 (c) 6 (d) –12

9. On subtracting 6 from –5, we get

(a) 1 (b) 11 (c) –11 (d) none of these

10. On subtracting –13 from –8, we get

(a) –21 (b) 21 (c) 5 (d) –5

11. $(-36) \div (-9) = ?$

(a) 4 (b) –4 (c) none of these

12. $0 \div (-5) = ?$

(a) –5 (b) 0 (c) not defined

13. $(-8) \div 0 = ?$

(a) –8 (b) 0 (c) not defined

14. Which of the following is a true statement?

(a) $-11 > -8$ (b) $-11 < -8$ (c) –11 and –8 cannot be compared

15. The sum of two integers is 6. If one of them is –3, then the other is

(a) –9 (b) 9 (c) 3 (d) –3

16. The sum of two integers is –4. If one of them is 6, then the other is

(a) –10 (b) 10 (c) 2 (d) –2

17. The sum of two integers is 14. If one of them is –8, then the other is

(a) 22 (b) –22 (c) 6 (d) –6

18. The additive inverse of –6 is

(a) $\frac{1}{6}$ (b) $-\frac{1}{6}$ (c) 6 (d) 5

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19. $(-15)\times 8+(-15)\times 2=?$

(a) 150 (b) −150 (c) 90 (d) −90

20. $(-12)\times 6-(-12)\times 4=?$

(a) 24 (b) −24 (c) 120 (d) −120

21. $(-27)\times(-16)+(-27)\times(-14)=?$

(a) −810 (b) 810 (c) −54 (d) 54

22. $30\times(-23)+30\times 14=?$

(a) −270 (b) 270 (c) 1110 (d) −1110

23. The sum of two integers is 93. If one of them is −59, the other one is

(a) 34 (b) −34 (c) 152 (d) −152

24. $(?)\div(-18)=-5$

(a) −90 (b) 90 (c) none of these

Things to Remember

1. *The numbers ..., −4, −3, −2, −1, 0, 1, 2, 3, 4, ..., are integers.*
2. *0 is an integer which is neither positive nor negative.*
3. *0 is less than every positive integer and greater than every negative integer.*
4. *If x and y are integers such that $x>y$ then $-x<-y$.*
   For example: $17>13$ *and* $-17<-13$.
5. *The absolute value of an integer is its numerical value regardless of its sign.*
   Thus, $|-7|=7$ *and* $|7|=7$. *Also,* $|0|=0$.
6. *To add two integers with like signs, we add their numerical values and give the sign of the addends to the sum.*
   Thus, $(-8)+(-7)=-15$ *and* $8+7=15$.
7. *To add two integers with unlike signs, we take the difference of their numerical values and give the sign of the integer having the greater absolute value to the difference.*
   Thus, $(-17)+9=-8$ *and* $17+(-8)=9$.
8. *For two integers a and b, we define* $a-b=a+(-b)$.
9. *To subtract an integer b from an integer a, we change the sign of b and add it to a.*
10. *All properties of operations on whole numbers are satisfied by these operations on integers.*
11. *If a and b are two integers then $(a-b)$ is also an integer.*
12. *$-a$ and a are negatives, or additive inverses of each other.*
13. *To find the product of two integers with like signs (i.e., both positives or both negatives), we multiply their numerical values and give a plus sign to the product.*
14. *To find the product of two integers with unlike signs (i.e., one positive and one negative), we multiply their numerical values and give a minus sign to the product.*
15. *The quotient of two negative or two positive integers is always positive.*
16. *The quotient of one positive and one negative integer is always negative.*

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TEST PAPER-1

A. **1.** The sum of two integers is –12. If one of them is 43, find the other.

2. The difference of an integer p and –8 is 3. Find the value of p.

3. Add the product of (–16) and (–9) to the quotient of (–132) by 6.

4. By what number should (–240) be divided to obtain 16?

5. What should be divided by (–7) to obtain 12?

6. Evaluate:

(i) $(-6) \times (-15) \times (-5)$ (ii) $(-8) \times (-5) \times 9$

(iii) $9 \times (-12) \times 10$ (iv) $(-75) \times 8$

(v) $(-5) \times (-5) \times (-5)$ ...... taken 5 times (vi) $(-1) \times (-1) \times (-1) \times$ ...... taken 25 times

7. Evaluate:

(i) $(-16) \times 12 + (-16) \times 8$ (ii) $25 \times (-33) + 25 \times (-17)$

(iii) $(-19) \times (-25) + (-19) \times (-15)$ (iv) $(-47) \times 68 - (-47) \times 38$

(v) $(-105) \div 21$ (vi) $(-168) \div (-14)$

(vii) $0 \div (-34)$ (viii) $37 \div 0$

B. ***Mark (✓) against the correct answer in each of the following:***

8. The sum of two integers is –6. If one of them is 2, then the other is

(a) –4 (b) 4 (c) 8 (d) –8

9. What must be subtracted from –7 to obtain –15?

(a) –8 (b) 8 (c) –22 (d) 22

10. $(?) \div (-18) = -6$

(a) –108 (b) 108 (c) 3 (d) none of these

11. $(-37) \times (-7) + (-37) \times (-3) = ?$

(a) 370 (b) –370 (c) 148 (d) –148

12. $(-25) \times 8 + (-25) \times 2 = ?$

(a) 250 (b) 150 (c) –250 (d) –150

13. $(-9) - (-6) = ?$

(a) –15 (b) –3 (c) 3 (d) 15

14. How much less than –2 is –8?

(a) 6 (b) –6 (c) 10 (d) –10

C. **15.** ***Fill in the blanks.***

(i) $(-35) \times \ldots = 35$ (ii) $(-53) \times (\ldots) = -53$

(iii) $(-14) \times (\ldots) = (-16) \times (-14)$ (iv) $(-21) \times (\ldots) = 0$

(v) $(-119) \div 17 = (\ldots)$ (vi) $(-247) \div (\ldots) = 13$

(vii) $(\ldots) \div 31 = 0$ (viii) $(\ldots) \div (-19) = -8$

D. **16.** ***Write 'T' for true and 'F' for false for each of the following:***

(i) $0 \div (-16) = 0$ (ii) $(-8) \div 0 = 0$

(iii) $(-1) \div (-1) = -1$ (iv) $(-36) \div (-1) = 36$

(v) $(-52) \div 13 = -4$ (vi) $68 \div (-17) = 4$

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2 Fractions

In class VI, we read about fractions, addition and subtraction of fractions, etc. We shall review these concepts in this chapter and take up multiplication and division of fractions.

Fractions *The numbers of the form* $\frac{a}{b}$, *where a and b are natural numbers, are known as fractions.*

In $\frac{a}{b}$, we call a as *numerator* and b as *denominator.*

EXAMPLES (i) $\frac{3}{5}$ is a fraction in which numerator = 3 and denominator = 5.

(ii) $\frac{17}{6}$ is a fraction in which numerator = 17 and denominator = 6.

(iii) $\frac{8}{1}$ is a fraction in which numerator = 8 and denominator = 1.

VARIOUS TYPES OF FRACTIONS

(i) Decimal fraction: *A fraction whose denominator is any of the numbers* 10, 100, 1000, *etc., is called a decimal fraction.*

EXAMPLES Each of the fractions $\frac{3}{10}, \frac{27}{100}, \frac{31}{1000}$, etc., is a decimal fraction.

(ii) Vulgar fraction: *A fraction whose denominator is a whole number, other than* 10,100, 1000, *etc., is called a vulgar fraction.*

EXAMPLES $\frac{2}{9}, \frac{4}{13}, \frac{13}{20}, \frac{27}{109}$, etc., are all vulgar fractions.

(iii) Proper fraction: *A fraction whose numerator is less than its denominator, is called a proper fraction.*

EXAMPLES $\frac{3}{7}, \frac{5}{11}, \frac{23}{40}, \frac{73}{100}$, etc., are all proper fractions.

(iv) Improper fraction: *A fraction whose numerator is more than or equal to its denominator, is called an improper fraction.*

EXAMPLES $\frac{11}{7}, \frac{25}{12}, \frac{41}{36}, \frac{53}{53}$, etc., are all improper fractions.

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(v) Mixed fraction: *A number which can be expressed as the sum of a natural number and a proper fraction, is called a mixed fraction.*

EXAMPLES $1\frac{3}{4}, 4\frac{5}{7}, 7\frac{9}{13}, 12\frac{6}{25}$, etc., are all mixed fractions.

EXAMPLE 1. *Convert each of the following into an improper fraction:*

(i) $1\frac{3}{4}$ *(ii)* $4\frac{5}{7}$ *(iii)* $7\frac{9}{13}$ *(iv)* $12\frac{6}{25}$

Solution We have:

(i) $1\frac{3}{4} = \frac{1 \times 4 + 3}{4} = \frac{7}{4}$.

(ii) $4\frac{5}{7} = \frac{4 \times 7 + 5}{4} = \frac{33}{7}$.

(iii) $7\frac{9}{13} = \frac{7 \times 13 + 9}{13} = \frac{100}{13}$.

(iv) $12\frac{6}{25} = \frac{12 \times 25 + 6}{25} = \frac{306}{25}$.

EXAMPLE 2. *Convert each of the following into a mixed fraction:*

(i) $\frac{38}{7}$ *(ii)* $\frac{47}{15}$ *(iii)* $\frac{189}{16}$

Solution (i) On dividing 38 by 7, we get quotient = 5 and remainder = 3.

$\therefore \quad \frac{38}{7} = 5\frac{3}{7}$.

7)38(5
−35
3

(ii) On dividing 47 by 15, we get quotient = 3 and remainder = 2.

$\therefore \quad \frac{47}{15} = 3\frac{2}{15}$.

15)47(3
−45
2

(iii) On dividing 189 by 16, we get quotient = 11 and remainder = 13.

$\therefore \quad \frac{189}{16} = 11\frac{13}{16}$.

16)189(11
−16
29
−16
13

An Important Property If the numerator and the denominator of a fraction are both multiplied by the same nonzero number, then its value is not changed.

Thus, $\frac{3}{4} = \frac{3 \times 2}{4 \times 2} = \frac{3 \times 3}{4 \times 3} = \frac{3 \times 4}{4 \times 4}$, etc.

(vi) Equivalent fractions: *A given fraction and the fraction obtained by multiplying (or dividing) its numerator and denominator by the same nonzero number, are called equivalent fractions.*

Thus, $\frac{3}{4}, \frac{6}{8}, \frac{9}{12}, \frac{12}{16}$, etc., are all equivalent fractions.

(vii) Like fractions: *Fractions having the same denominator but different numerators are called like fractions.*

EXAMPLES $\frac{5}{14}, \frac{9}{14}, \frac{11}{14}$, etc., are like fractions.

(viii) Unlike fractions: *Fractions having different denominators are called unlike fractions.*

EXAMPLES $\frac{2}{5}, \frac{5}{7}, \frac{9}{13}$, etc., are unlike fractions.

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METHOD OF CHANGING UNLIKE FRACTIONS TO LIKE FRACTIONS

Step 1. *Find the LCM of the denominators of all the given fractions.*

Step 2. *Change each of the given fractions into an equivalent fraction having denominator equal to the LCM of the denominators of the given fractions.*

EXAMPLE 3. *Convert the fractions* $\frac{5}{6}, \frac{7}{9}, \frac{11}{12}$ *into like fractions.*

Solution LCM of 6, 9, 12 = $(3 \times 2 \times 3 \times 2) = 36$.

Now, $\frac{5}{6} = \frac{5 \times 6}{6 \times 6} = \frac{30}{36}$; $\frac{7}{9} = \frac{7 \times 4}{9 \times 4} = \frac{28}{36}$ and $\frac{11}{12} = \frac{11 \times 3}{12 \times 3} = \frac{33}{36}$.

Clearly, $\frac{30}{36}, \frac{28}{36}, \frac{33}{36}$ are like fractions.

$$\begin{array}{r|l} 3 & 6-9-12 \\ \hline 2 & 2-3-4 \\ \hline & 1-3-2 \end{array}$$

(ix) Irreducible fractions: *A fraction* $\frac{a}{b}$ *is said to be irreducible or in lowest terms, if HCF of a and b is 1.*

If HCF of a and b is other than 1 then $\frac{a}{b}$ *is said to be reducible.*

EXAMPLE 4. *Convert* $\frac{84}{98}$ *into irreducible form.*

Solution First we find the HCF of 84 and 98.

Clearly, HCF of 84 and 98 is 14.

So, we divide the numerator and denominator of the given fraction by 14.

$\therefore \quad \frac{84}{98} = \frac{84 \div 14}{98 \div 14} = \frac{6}{7}$.

Hence, $\frac{84}{98}$ in irreducible form is $\frac{6}{7}$.

$$\begin{array}{l} 84)\overline{98}(1 \\ \quad -84 \\ \quad \overline{14)84}(6 \\ \qquad -84 \\ \qquad \overline{\times} \end{array}$$

COMPARING FRACTIONS

Let $\frac{a}{b}$ and $\frac{c}{d}$ be two given fractions. Then,

(i) $\frac{a}{b} > \frac{c}{d} \Leftrightarrow ad > bc$ (ii) $\frac{a}{b} = \frac{c}{d} \Leftrightarrow ad = bc$ (iii) $\frac{a}{b} < \frac{c}{d} \Leftrightarrow ad < bc$.

EXAMPLE 5. *Compare the fractions:* (i) $\frac{3}{5}, \frac{5}{8}$ (ii) $\frac{9}{16}, \frac{13}{24}$.

Solution (i) By cross multiplication, we have:

$3 \times 8 = 24$ and $5 \times 5 = 25$.

But, $24 < 25$.

$\therefore \quad \frac{3}{5} < \frac{5}{8}$.

$$\begin{array}{ccc} 3 & \searrow\!\!\nearrow & 5 \\ 5 & \nearrow\!\!\searrow & 8 \end{array}$$

(ii) By cross multiplication, we have:

$9 \times 24 = 216$ and $16 \times 13 = 208$.

But, $216 > 208$

$\therefore \quad \frac{9}{16} > \frac{13}{24}$.

$$\begin{array}{ccc} 9 & \searrow\!\!\nearrow & 13 \\ 16 & \nearrow\!\!\searrow & 24 \end{array}$$

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METHOD OF COMPARING MORE THAN TWO FRACTIONS

Step 1. *Find the LCM of the denominators of the given fractions. Let it be m.*

Step 2. *Convert all the given fractions into like fractions, each having m as denominator.*

Step 3. *Now, if we compare any two of these like fractions, then the one having larger numerator is larger.*

EXAMPLE 6. *Arrange the fractions* $\frac{2}{5}, \frac{3}{10}, \frac{9}{14}, \frac{16}{35}$ *in ascending order.*

Solution The given fractions are $\frac{2}{5}, \frac{3}{10}, \frac{9}{14}, \frac{16}{35}$.

LCM of 5, 10, 14, 35 = $(5 \times 2 \times 7) = 70$.

5	5—10—14—35
2	1—2—14—7
7	1—1—7—7
	1—1—1—1

Now, let us change each of the given fractions into an equivalent fraction having 70 as its denominator.

Now, $\frac{2}{5} = \frac{2 \times 14}{5 \times 14} = \frac{28}{70}; \frac{3}{10} = \frac{3 \times 7}{10 \times 7} = \frac{21}{70}; \frac{9}{14} = \frac{9 \times 5}{14 \times 5} = \frac{45}{70}$

and $\frac{16}{35} = \frac{16 \times 2}{35 \times 2} = \frac{32}{70}$.

Clearly, $\frac{21}{70} < \frac{28}{70} < \frac{32}{70} < \frac{45}{70}$.

Hence, $\frac{3}{10} < \frac{2}{5} < \frac{16}{35} < \frac{9}{14}$.

Hence, the given fractions in ascending order are $\frac{3}{10}, \frac{2}{5}, \frac{16}{35}, \frac{9}{14}$.

ADDITION AND SUBTRACTION OF FRACTIONS

ADDITION OF FRACTIONS

Rule 1. *For adding two like fractions, the numerators are added and the denominator remains the same.*

EXAMPLES (i) $\frac{2}{9} + \frac{5}{9} = \frac{2+5}{9} = \frac{7}{9}$. (ii) $\frac{8}{15} + \frac{6}{15} = \frac{8+6}{15} = \frac{14}{15}$.

Rule 2. *For addition of two unlike fractions, first change them to like fractions and then add them as given in Rule 1.*

EXAMPLE 7. *Add:* $\frac{3}{10} + \frac{8}{15}$.

Solution LCM of 10 and 15 = $(5 \times 2 \times 3) = 30$.

5	10—15
	2—3

$\therefore \quad \frac{3}{10} = \frac{3 \times 3}{10 \times 3} = \frac{9}{30}$ and $\frac{8}{15} = \frac{8 \times 2}{15 \times 2} = \frac{16}{30}$.

$\therefore \quad \frac{3}{10} + \frac{8}{15} = \frac{9}{30} + \frac{16}{30} = \frac{9+16}{30} = \frac{\cancel{25}^{5}}{\cancel{30}_{6}} = \frac{5}{6}$.

Short Cut Method:

$\frac{3}{10} + \frac{8}{15} = \frac{9+16}{30} = \frac{\cancel{25}^{5}}{\cancel{30}_{6}} = \frac{5}{6}$.

$\begin{bmatrix} 30 \div 10 = 3 \text{ and } 3 \times 3 = 9 \\ 30 \div 15 = 2 \text{ and } 2 \times 8 = 16 \end{bmatrix}$

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EXAMPLE 8. *Find the sum:* $\frac{13}{14}+\frac{27}{35}$.

Solution LCM of 14, 35 = $(7\times2\times5)=70$.

$$\frac{13}{14}+\frac{27}{35}=\frac{65+54}{70}$$

$$=\frac{119}{70}=\frac{17}{10}=1\frac{7}{10}.$$

$\begin{bmatrix}70\div14=5 \text{ and } 5\times13=65\\ 70\div35=2 \text{ and } 2\times27=54\end{bmatrix}$

$\begin{array}{r|l} 7 & 14-35 \\ \hline & 2-5 \end{array}$

Properties of Addition of Fractions:

(i) *Addition of fractions is associative, i.e.,* $\left(\frac{a}{b}+\frac{c}{d}\right)+\frac{e}{f}=\frac{a}{b}+\left(\frac{c}{d}+\frac{e}{f}\right)$.

(ii) *Addition of fractions is commutative, i.e.,* $\left(\frac{a}{b}+\frac{c}{d}\right)=\left(\frac{c}{d}+\frac{a}{b}\right)$.

SUBTRACTION OF FRACTIONS

The subtraction of fractions can be performed in a manner similar to that of addition.

EXAMPLE 9. *Find the difference:*

(i) $\frac{7}{9}-\frac{5}{9}$ (ii) $\frac{13}{16}-\frac{7}{12}$ (iii) $\frac{11}{15}-\frac{9}{20}$

Solution We have:

(i) $\frac{7}{9}-\frac{5}{9}=\frac{(7-5)}{9}=\frac{2}{9}$.

(ii) LCM of 16 and 12 = $(4\times4\times3)=48$.

$\therefore \frac{13}{16}-\frac{7}{12}=\frac{(39-28)}{48}=\frac{11}{48}$.

$\begin{bmatrix}48\div16=3 \text{ and } 3\times13=39\\ 48\div12=4 \text{ and } 4\times7=28\end{bmatrix}$

$\begin{array}{r|l} 4 & 16-12 \\ \hline & 4-3 \end{array}$

(iii) LCM of 15 and 20 = $(5\times3\times4)=60$.

$\therefore \frac{11}{15}-\frac{9}{20}=\frac{(44-27)}{60}=\frac{17}{60}$.

$\begin{bmatrix}60\div15=4 \text{ and } 4\times11=44\\ 60\div20=3 \text{ and } 3\times9=27\end{bmatrix}$

$\begin{array}{r|l} 5 & 15-20 \\ \hline & 3-4 \end{array}$

EXAMPLE 10. *Simplify:* $\frac{5}{9}-\frac{7}{12}+\frac{1}{2}$.

Solution LCM of 9, 12, 2 = $(2\times3\times3\times2)=36$.

$$\therefore \frac{5}{9}-\frac{7}{12}+\frac{1}{2}=\frac{(20-21+18)}{36}$$

$$=\frac{(38-21)}{36}=\frac{17}{36}.$$

$\begin{bmatrix}36\div9=4 \text{ and } 4\times5=20\\ 36\div12=3 \text{ and } 3\times7=21\\ 36\div2=18 \text{ and } 18\times1=18\end{bmatrix}$

$\begin{array}{r|l} 2 & 9-12-2 \\ \hline 3 & 9-6-1 \\ \hline & 3-2-1 \end{array}$

EXAMPLE 11. *Simplify:* $3\frac{1}{5}+2\frac{1}{10}-1\frac{1}{2}-\frac{1}{4}$.

Solution We have:

$$3\frac{1}{5}+2\frac{1}{10}-1\frac{1}{2}-\frac{1}{4}=\frac{16}{5}+\frac{21}{10}-\frac{3}{2}-\frac{1}{4}$$

$$=\frac{(64+42-30-5)}{20}$$

[$\because$ LCM of 5, 10, 2, 4 = $2\times5\times2$]

$$=\frac{(106-35)}{20}=\frac{71}{20}=3\frac{11}{20}.$$

$\begin{array}{r|l} 2 & 5-10-2-4 \\ \hline 5 & 5-5-1-2 \\ \hline & 1-1-1-2 \end{array}$

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EXAMPLE 12. *Sarita bought $5\frac{3}{4}$ kg potatoes and $3\frac{1}{2}$ kg tomatoes from a vendor. What is the total weight of vegetables bought by her?*

Solution Total weight of vegetables bought by Sarita

$$= \left(\frac{23}{4} + \frac{7}{2}\right) \text{kg} = \frac{(23+14)}{4} \text{ kg}$$

$$= \frac{37}{4} \text{ kg} = 9\frac{1}{4} \text{ kg.}$$

EXAMPLE 13. *Rohit ate $\frac{4}{7}$ part of an apple and his sister Ritu ate the remaining part of it? Who ate more and by how much?*

Solution Part of apple eaten by Rohit $= \frac{4}{7}$.

Remaining part of apple $= \left(1 - \frac{4}{7}\right) = \frac{(7-4)}{7} = \frac{3}{7}$.

$\therefore$ part of apple eaten by Ritu $= \frac{3}{7}$.

Clearly, $\frac{4}{7} > \frac{3}{7}$.

So, Rohit ate more.

Difference of their parts $= \left(\frac{4}{7} - \frac{3}{7}\right) = \frac{(4-3)}{7} = \frac{1}{7}$.

EXAMPLE 14. *What should be added to $15\frac{2}{3}$ to get $18\frac{5}{6}$?*

Solution Required number to be added $= \left(18\frac{5}{6} - 15\frac{2}{3}\right) = \left(\frac{113}{6} - \frac{47}{3}\right)$

$$= \frac{(113-94)}{6} = \frac{19}{6} = 3\frac{1}{6}.$$

EXAMPLE 15. *What should be subtracted from $17\frac{3}{4}$ to get $11\frac{2}{3}$?*

Solution Required number to be subtracted $= \left(17\frac{3}{4} - 11\frac{2}{3}\right) = \left(\frac{71}{4} - \frac{35}{3}\right)$

$$= \frac{(213-140)}{12} = \frac{73}{12} = 6\frac{1}{12}.$$

EXERCISE 2A

1. Compare the fractions:

 (i) $\frac{5}{8}$ and $\frac{7}{12}$ (ii) $\frac{5}{9}$ and $\frac{11}{15}$ (iii) $\frac{11}{12}$ and $\frac{15}{16}$

2. Arrange the following fractions in ascending order:

 (i) $\frac{3}{4}, \frac{5}{6}, \frac{7}{9}, \frac{11}{12}$ (ii) $\frac{4}{5}, \frac{7}{10}, \frac{11}{15}, \frac{17}{20}$

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3. Arrange the following fractions in descending order:

(i) $\frac{3}{4}, \frac{7}{8}, \frac{7}{12}, \frac{17}{24}$ (ii) $\frac{2}{3}, \frac{3}{5}, \frac{7}{10}, \frac{8}{15}$

4. Reenu got $\frac{2}{7}$ part of an apple while Sonal got $\frac{4}{5}$ part of it. Who got the larger part and by how much?

5. Find the sum:

(i) $\frac{5}{9}+\frac{3}{9}$ (ii) $\frac{8}{9}+\frac{7}{12}$ (iii) $\frac{5}{6}+\frac{7}{8}$

(iv) $\frac{7}{12}+\frac{11}{16}+\frac{9}{24}$ (v) $3\frac{4}{5}+2\frac{3}{10}+1\frac{1}{15}$ (vi) $8\frac{3}{4}+10\frac{2}{5}$

6. Find the difference:

(i) $\frac{5}{7}-\frac{2}{7}$ (ii) $\frac{5}{6}-\frac{3}{4}$ (iii) $3\frac{1}{5}-\frac{7}{10}$

(iv) $7-4\frac{2}{3}$ (v) $3\frac{3}{10}-1\frac{7}{15}$ (vi) $2\frac{5}{9}-1\frac{7}{15}$

7. Simplify:

(i) $\frac{2}{3}+\frac{5}{6}-\frac{1}{9}$ (ii) $8-4\frac{1}{2}-2\frac{1}{4}$ (iii) $8\frac{5}{6}-3\frac{3}{8}+1\frac{7}{12}$

8. Aneeta bought $3\frac{3}{4}$ kg apples and $4\frac{1}{2}$ kg guava. What is the total weight of fruits purchased by her?

9. A rectangular sheet of paper is $15\frac{3}{4}$ cm long and $12\frac{1}{2}$ cm wide. Find its perimeter.

10. A picture is $7\frac{3}{5}$ cm wide. How much should it be trimmed to fit in a frame $7\frac{3}{10}$ cm wide?

11. What should be added to $7\frac{3}{5}$ to get 18?

12. What should be added to $7\frac{4}{15}$ to get $8\frac{2}{5}$?

13. A piece of wire $3\frac{3}{4}$ m long broke into two pieces. One piece is $1\frac{1}{2}$ m long. How long is the other piece?

14. A film show lasted for $3\frac{2}{3}$ hours. Out of this time $1\frac{1}{2}$ hours was spent on advertisements. What was the actual duration of the film?

15. Of $\frac{2}{3}$ and $\frac{5}{9}$, which is greater and by how much?

16. The cost of a pen is ₹$16\frac{3}{5}$ and that of a pencil is ₹ $4\frac{3}{4}$. Which costs more and by how much?

MULTIPLICATION OF FRACTIONS

Rule: *Product of Fractions* $= \frac{\textit{Product of their Numerators}}{\textit{Product of their Denominators}}$.

Thus, $\left(\frac{a}{b}\times\frac{c}{d}\right)=\frac{(a\times c)}{(b\times d)}$.

EXAMPLE 1. Find the product:

(i) $\frac{5}{8}\times\frac{3}{4}$ *(ii)* $\frac{3}{4}\times\frac{5}{2}$ *(iii)* $\frac{9}{16}\times\frac{8}{15}$ *(iv)* $\frac{5}{12}\times 9$

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Solution We have:

(i) $\frac{5}{8}\times\frac{3}{4}=\frac{5\times3}{8\times4}=\frac{15}{32}$.

(ii) $\frac{3}{4}\times\frac{5}{2}=\frac{3\times5}{4\times2}=\frac{15}{8}=1\frac{7}{8}$.

(iii) $\frac{9}{16}\times\frac{8}{15}=\frac{\cancel{9}^{3}\times\cancel{8}^{1}}{\cancel{16}_{2}\times\cancel{15}_{5}}=\frac{3}{10}$.

(iv) $\frac{5}{12}\times9=\frac{5}{12}\times\frac{9}{1}=\frac{5\times\cancel{9}^{3}}{\cancel{12}_{4}\times1}=\frac{15}{4}=3\frac{3}{4}$.

EXAMPLE 2. *Multiply:*

(i) $7\frac{5}{9}$ *by* $\frac{3}{2}$ (ii) $9\frac{3}{8}$ *by* 12 (iii) $6\frac{11}{14}$ *by* $3\frac{1}{2}$

Solution We have:

(i) $7\frac{5}{9}\times\frac{3}{2}=\frac{68}{9}\times\frac{3}{2}=\frac{\cancel{68}^{34}\times\cancel{3}^{1}}{\cancel{9}_{3}\times\cancel{2}_{1}}=\frac{34}{3}=11\frac{1}{3}$.

(ii) $9\frac{3}{8}\times12=\frac{75}{\cancel{8}_{2}}\times\frac{\cancel{12}^{3}}{1}=\frac{225}{2}=112\frac{1}{2}$.

(iii) $6\frac{11}{14}\times3\frac{1}{2}=\frac{95}{14}\times\frac{7}{2}=\frac{95\times\cancel{7}^{1}}{\cancel{14}_{2}\times2}=\frac{95}{4}=23\frac{3}{4}$.

EXAMPLE 3. *Simplify:* $\frac{14}{25}\times\frac{35}{51}\times\frac{34}{49}$.

Solution We have:

$$\frac{14}{25}\times\frac{35}{51}\times\frac{34}{49}=\frac{\cancel{14}^{2}\times\cancel{35}^{\cancel{5}1}\times\cancel{34}^{2}}{\cancel{25}_{5}\times\cancel{51}_{3}\times\cancel{49}_{\cancel{7}1}}=\frac{4}{15}.$$

EXAMPLE 4. *Simplify:* $3\frac{4}{7}\times2\frac{2}{5}\times1\frac{3}{4}$.

Solution We have:

$$3\frac{4}{7}\times2\frac{2}{5}\times1\frac{3}{4}=\frac{25}{7}\times\frac{12}{5}\times\frac{7}{4}=\frac{\cancel{25}^{5}\times\cancel{12}^{3}\times\cancel{7}^{1}}{\cancel{7}_{1}\times\cancel{5}_{1}\times\cancel{4}_{1}}=15.$$

Use of 'OF'

We define: $\frac{a}{b}$ of $c=\left(c\times\frac{a}{b}\right)$.

EXAMPLE 5. *Find:*

(i) $\frac{2}{5}$ *of* 40 (ii) $\frac{5}{9}$ *of* 48 (iii) $\frac{11}{14}$ *of* 63

Solution We have:

(i) $\frac{2}{5}$ of $40=\frac{2}{5}$ of $\frac{40}{1}=\frac{40}{1}\times\frac{2}{5}=\frac{\cancel{40}^{8}\times2}{1\times\cancel{5}_{1}}=16$.

(ii) $\frac{5}{9}$ of $48=\frac{5}{9}$ of $\frac{48}{1}=\frac{48}{1}\times\frac{5}{9}=\frac{\cancel{48}^{16}\times5}{1\times\cancel{9}_{3}}=\frac{80}{3}=26\frac{2}{3}$.

(iii) $\frac{11}{14}$ of $63=\frac{11}{14}$ of $\frac{63}{1}=\frac{63}{1}\times\frac{11}{14}=\frac{\cancel{63}^{9}\times11}{1\times\cancel{14}_{2}}=\frac{99}{2}=49\frac{1}{2}$.

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EXAMPLE 6. *Find:*

(i) $\frac{1}{5}$ *of a rupee* (ii) $\frac{2}{3}$ *of an year* (iii) $\frac{5}{8}$ *of a day*

(iv) $\frac{7}{8}$ *of a kilogram* (v) $\frac{11}{25}$ *of a litre* (vi) $\frac{4}{5}$ *of an hour*

Solution We have:

(i) $\frac{1}{5}$ of a rupee $= \frac{1}{5}$ of 100 paise $= \left(100 \times \frac{1}{5}\right)$ paise

$= \left(\frac{100}{1} \times \frac{1}{5}\right)$ paise $= \left(\frac{\cancel{100}^{20} \times 1}{1 \times \cancel{5}_1}\right)$ paise $= 20$ paise.

(ii) $\frac{2}{3}$ of an year $= \frac{2}{3}$ of 12 months $= \left(12 \times \frac{2}{3}\right)$ months

$= \left(\frac{12}{1} \times \frac{2}{3}\right)$ months $= \frac{(\cancel{12}^4 \times 2)}{(1 \times \cancel{3}_1)}$ months $= 8$ months.

(iii) $\frac{5}{8}$ of a day $= \frac{5}{8}$ of 24 hours $= \left(24 \times \frac{5}{8}\right)$ hours

$= \left(\frac{24}{1} \times \frac{5}{8}\right)$ hours $= \left(\frac{\cancel{24}^3 \times 5}{1 \times \cancel{8}_1}\right)$ hours $= 15$ hours.

(iv) $\frac{7}{8}$ of a kilogram $= \frac{7}{8}$ of 1000 g $= \left(1000 \times \frac{7}{8}\right)$ g

$= \left(\frac{1000}{1} \times \frac{7}{8}\right)$ g $= \left(\frac{\cancel{1000}^{125} \times 7}{1 \times \cancel{8}_1}\right)$ g $= 875$ g.

(v) $\frac{11}{25}$ of a litre $= \frac{11}{25}$ of 1000 mL $= \left(1000 \times \frac{11}{25}\right)$ mL

$= \left(\frac{1000}{1} \times \frac{11}{25}\right)$ mL $= \left(\frac{\cancel{1000}^{40} \times 11}{1 \times \cancel{25}_1}\right)$ mL $= 440$ mL.

(vi) $\frac{4}{5}$ of an hour $= \frac{4}{5}$ of 60 min $= \left(60 \times \frac{4}{5}\right)$ min

$= \left(\frac{60}{1} \times \frac{4}{5}\right)$ min $= \left(\frac{\cancel{60}^{12} \times 4}{1 \times \cancel{5}_1}\right)$ min $= 48$ min.

EXAMPLE 7. *Milk is sold at ₹ $16\frac{3}{4}$ per litre. Find the cost of $6\frac{2}{5}$ litres of milk.*

Solution Cost of 1 litre of milk $=$ ₹ $16\frac{3}{4} =$ ₹ $\frac{67}{4}$.

Cost of $6\frac{2}{5}$ litres of milk $=$ ₹ $\left(\frac{67}{4} \times \frac{32}{5}\right)$

$=$ ₹ $\left(\frac{67 \times \cancel{32}^8}{\cancel{4}_1 \times 5}\right) =$ ₹ $\left(\frac{536}{5}\right) =$ ₹ $107\frac{1}{5}$.

Hence, the cost of $6\frac{2}{5}$ litres of milk is ₹ $107\frac{1}{5}$.

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EXAMPLE 8. *Sajal can walk* $2\frac{2}{5}$ km *in an hour. How much distance will he cover in* $3\frac{1}{3}$ *hours?*

Solution Distance covered by Sajal in 1 hour = $2\frac{2}{5}$ km = $\frac{12}{5}$ km.

Distance covered by Sajal in $3\frac{1}{3}$ hours = $\left(\frac{12}{5} \times \frac{10}{3}\right)$ km

$$= \left(\frac{\cancel{12}^{4} \times \cancel{10}^{2}}{\cancel{5}_{1} \times \cancel{3}_{1}}\right) \text{km} = 8 \text{ km}.$$

Hence, the distance covered by Sajal in 1 hour is 8 km.

EXAMPLE 9. *A carton contains* 16 *boxes of nails and each box weighs* $4\frac{3}{4}$ kg. How much would a carton of nails weigh?

Solution Weight of 1 box = $4\frac{3}{4}$ kg = $\frac{19}{4}$ kg.

Weight of 16 boxes = $\left(\frac{19}{4} \times 16\right)$ kg = $\left(\frac{19}{4} \times \frac{16}{1}\right)$ kg

$$= \left(\frac{19 \times \cancel{16}^{4}}{\cancel{4}_{1} \times 1}\right) \text{kg} = 76 \text{ kg}.$$

Hence, the weight of a carton is 76 kg.

EXAMPLE 10. *A book consists of* 216 *pages. During last week Vikas read* $\frac{3}{4}$ *of the book. How many pages did he read?*

Solution Total number of pages in the book = 216.

Number of pages read = $\left(\frac{3}{4} \text{ of } 216\right) = \left(216 \times \frac{3}{4}\right)$

$$= \left(\frac{216}{1} \times \frac{3}{4}\right) = \left(\frac{\cancel{216}^{54} \times 3}{1 \times \cancel{4}_{1}}\right) = 162.$$

Hence, Vikas read 162 pages during last week.

EXAMPLE 11. *A tin contains* 18 kg *ghee. After consuming* $\frac{2}{3}$ *of it, how much ghee is left in the tin?*

Solution Total quantity of ghee in the tin = 18 kg.

Quantity of ghee consumed = $\frac{2}{3}$ of 18 kg = $\left(18 \times \frac{2}{3}\right)$ kg

$$= \left(\frac{18}{1} \times \frac{2}{3}\right) \text{kg} = \left(\frac{\cancel{18}^{6} \times 2}{1 \times \cancel{3}_{1}}\right) \text{kg} = 12 \text{ kg}.$$

Quantity of ghee left in the tin = (18 − 12) kg = 6 kg.

EXAMPLE 12. *Renu spends* $\frac{4}{5}$ *of her income on household expenses. Her monthly income is* ₹ 30000. *How much does she save every month?*

Solution Total monthly income = ₹ 30000.

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Monthly expenditure $= \frac{4}{5}$ of ₹ 30000

$= ₹\left(30000 \times \frac{4}{5}\right) = ₹\left(\frac{30000}{1} \times \frac{4}{5}\right)$

$= ₹\left(\frac{\overset{6000}{\cancel{30000}} \times 4}{1 \times \cancel{5}_1}\right) = ₹24000.$

Monthly savings = ₹ (30000 − 24000) = ₹ 6000.

EXERCISE 2B

1. Find the product:

(i) $\frac{3}{5} \times \frac{7}{11}$ (ii) $\frac{5}{8} \times \frac{4}{7}$ (iii) $\frac{4}{9} \times \frac{15}{16}$

(iv) $\frac{2}{5} \times 15$ (v) $\frac{8}{15} \times 20$ (vi) $\frac{5}{8} \times 1000$

(vii) $3\frac{1}{8} \times 16$ (viii) $2\frac{4}{15} \times 12$ (ix) $3\frac{6}{7} \times 4\frac{2}{3}$

(x) $9\frac{1}{2} \times 1\frac{9}{19}$ (xi) $4\frac{1}{8} \times 2\frac{10}{11}$ (xii) $5\frac{5}{6} \times 1\frac{5}{7}$

2. Simplify:

(i) $\frac{2}{3} \times \frac{5}{44} \times \frac{33}{35}$ (ii) $\frac{12}{25} \times \frac{15}{28} \times \frac{35}{36}$ (iii) $\frac{10}{27} \times \frac{28}{65} \times \frac{39}{56}$

(iv) $1\frac{4}{7} \times 1\frac{13}{22} \times 1\frac{1}{15}$ (v) $2\frac{2}{17} \times 7\frac{2}{9} \times 1\frac{33}{52}$ (vi) $3\frac{1}{16} \times 7\frac{3}{7} \times 1\frac{25}{39}$

3. Find:

(i) $\frac{1}{3}$ of 24 (ii) $\frac{3}{4}$ of 32 (iii) $\frac{5}{9}$ of 45

(iv) $\frac{7}{50}$ of 1000 (v) $\frac{3}{20}$ of 1020 (vi) $\frac{5}{11}$ of ₹ 220

(vii) $\frac{4}{9}$ of 54 metres (viii) $\frac{6}{7}$ of 35 litres (ix) $\frac{1}{6}$ of an hour

(x) $\frac{5}{6}$ of an year (xi) $\frac{7}{20}$ of a kg (xii) $\frac{9}{20}$ of a metre

(xiii) $\frac{7}{8}$ of a day (xiv) $\frac{3}{7}$ of a week (xv) $\frac{7}{50}$ of a litre

4. Apples are sold at ₹ $48\frac{4}{5}$ per kg. What is the cost of $3\frac{3}{4}$ kg of apples?

5. Cloth is being sold at ₹ $42\frac{1}{2}$ per metre. What is the cost of $5\frac{3}{5}$ metres of this cloth?

6. A car covers a certain distance at a uniform speed of $66\frac{2}{3}$ km per hour. How much distance will it cover in 9 hours?

7. One tin holds $12\frac{3}{4}$ litres of oil. How many litres of oil can 26 such tins hold?

8. For a particular show in a circus, each ticket costs ₹ $35\frac{1}{2}$. If 308 tickets are sold for the show, how much amount has been collected?

9. Nine boards are stacked on the top of each other. The thickness of each board is $3\frac{2}{3}$ cm. How high is the stack?

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10. Rohit takes $4\frac{4}{5}$ minutes to make a complete round of a circular park. How much time will he take to make 15 rounds?

11. Amit weighs 35 kg. His sister Kavita's weight is $\frac{3}{5}$ of Amit's weight. How much does Kavita weigh?

12. There are 42 students in a class and $\frac{5}{7}$ of the students are boys. How many girls are there in the class?

13. Sapna earns ₹ 24000 per month. She spends $\frac{7}{8}$ of her income and deposits rest of the money in a bank. How much money does she deposit in the bank each month?

14. Each side of a square field is $4\frac{2}{3}$ m. Find its area.

15. Find the area of a rectangular park which is $41\frac{2}{3}$ m long and $18\frac{3}{5}$ m broad.

DIVISION OF FRACTIONS

RECIPROCAL OF A FRACTION

Two fractions are said to be the reciprocal of each other, if their product is 1.

For example, $\frac{4}{9}$ and $\frac{9}{4}$ are the reciprocals of each other, since $\left(\frac{4}{9}\times\frac{9}{4}\right)=1$.

In general, if $\frac{a}{b}$ is a fraction, then its reciprocal is $\frac{b}{a}$.

Resiprocal of 0 does not exist.

EXAMPLE 1. *Write down the reciprocal of:*

(i) $\frac{3}{7}$ *(ii)* $\frac{1}{5}$ *(iii)* 6 *(iv)* $3\frac{5}{8}$

Solution We have:

(i) Reciprocal of $\frac{3}{7}$ is $\frac{7}{3}$. $\left[\because \frac{3}{7}\times\frac{7}{3}=1\right]$

(ii) Reciprocal of $\frac{1}{5}$ is $\frac{5}{1}=5$. $\left[\because \frac{1}{5}\times 5=1\right]$

(iii) Reciprocal of 6 is $\frac{1}{6}$. $\left[\because 6\times\frac{1}{6}=1\right]$

(iv) Reciprocal of $3\frac{5}{8}$ = reciprocal of $\frac{29}{8}=\frac{8}{29}$. $\left[\because \frac{29}{8}\times\frac{8}{29}=1\right]$

DIVISION OF FRACTIONS

Rule *To divide a fraction by another fraction, the first fraction is multiplied by the reciprocal of the second.*

Thus, $\left(\frac{a}{b}\div\frac{c}{d}\right)=\left(\frac{a}{b}\times\frac{d}{c}\right)$.

EXAMPLE 2. *Simplify:*

(i) $\frac{4}{9}\div\frac{5}{6}$ *(ii)* $\frac{5}{7}\div 10$ *(iii)* $5\frac{3}{5}\div 2\frac{1}{10}$

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Solution We have:

(i) $\frac{4}{9} \div \frac{5}{6} = \frac{4}{9} \times \frac{6}{5}$ $\left[\because \text{ the reciprocal of } \frac{5}{6} \text{ is } \frac{6}{5}\right]$

$= \frac{4 \times \cancel{6}^2}{\cancel{9}_3 \times 5} = \frac{8}{15}.$

(ii) $\frac{5}{7} \div 10 = \frac{5}{7} \div \frac{10}{1}$

$= \frac{5}{7} \times \frac{1}{10}$ $\left[\because \text{ the reciprocal of } \frac{10}{1} \text{ is } \frac{1}{10}\right]$

$= \frac{\cancel{5}^1 \times 1}{7 \times \cancel{10}_2} = \frac{1}{14}.$

(iii) $5\frac{3}{5} \div 2\frac{1}{10} = \frac{28}{5} \div \frac{21}{10}$

$= \frac{28}{5} \times \frac{10}{21}$ $\left[\because \text{ the reciprocal of } \frac{21}{10} \text{ is } \frac{10}{21}\right]$

$= \frac{\cancel{28}^4 \times \cancel{10}^2}{\cancel{5}_1 \times \cancel{21}_3} = \frac{8}{3} = 2\frac{2}{3}.$

EXAMPLE 3. *Divide:*

(i) $\frac{5}{9}$ *by* $\frac{2}{3}$ *(ii)* $5\frac{4}{7}$ *by* $\frac{13}{14}$ *(iii)* $4\frac{2}{7}$ *by* $2\frac{2}{5}$

Solution We have:

(i) $\frac{5}{9} \div \frac{2}{3} = \frac{5}{9} \times \frac{3}{2}$ $\left[\because \text{ reciprocal of } \frac{2}{3} \text{ is } \frac{3}{2}\right]$

$= \frac{5 \times \cancel{3}^1}{\cancel{9}_3 \times 2} = \frac{5}{6}.$

(ii) $5\frac{4}{7} \div \frac{13}{14} = \frac{39}{7} \div \frac{13}{14}$

$= \frac{39}{7} \times \frac{14}{13}$ $\left[\because \text{ reciprocal of } \frac{13}{14} \text{ is } \frac{14}{13}\right]$

$= \frac{\cancel{39}^3 \times \cancel{14}^2}{\cancel{7}_1 \times \cancel{13}_1} = \frac{6}{1} = 6.$

(iii) $4\frac{2}{7} \div 2\frac{2}{5} = \frac{30}{7} \div \frac{12}{5}$

$= \frac{30}{7} \times \frac{5}{12}$ $\left[\because \text{ reciprocal of } \frac{12}{5} \text{ is } \frac{5}{12}\right]$

$= \frac{\cancel{30}^5 \times 5}{7 \times \cancel{12}_2} = \frac{25}{14} = 1\frac{11}{14}.$

EXAMPLE 4. *Divide:*

(i) 28 *by* $\frac{7}{4}$ *(ii)* 36 *by* $6\frac{2}{3}$

Solution We have:

(i) $28 \div \frac{7}{4} = \frac{28}{1} \div \frac{7}{4} = \frac{28}{1} \times \frac{4}{7}$ $\left[\because \text{ reciprocal of } \frac{7}{4} \text{ is } \frac{4}{7}\right]$

$= \frac{\cancel{28}^4 \times 4}{1 \times \cancel{7}_1} = \frac{16}{1} = 16.$

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(ii) $36 \div 6\frac{2}{3} = \frac{36}{1} \div \frac{20}{3}$

$= \frac{36}{1} \times \frac{3}{20}$ $\left[\because \text{ reciprocal of } \frac{20}{3} \text{ is } \frac{3}{20}\right]$

$= \frac{36^{9} \times 3}{1 \times 20_{5}} = \frac{27}{5} = 5\frac{2}{5}$.

EXAMPLE 5. *A rope of length $9\frac{3}{4}$ m is cut into 6 pieces of equal length. Find the length of each piece.*

Solution Length of the rope $= 9\frac{3}{4}$ m $= \frac{39}{4}$ m.

Number of equal pieces = 6.

Length of each piece $= \left(\frac{39}{4} \div 6\right)$ m $= \left(\frac{39}{4} \div \frac{6}{1}\right)$ m

$= \left(\frac{39}{4} \times \frac{1}{6} \text{ m}\right)$ $\left[\because \text{ reciprocal of 6 is } \frac{1}{6}\right]$

$= \frac{39^{13} \times 1}{4 \times 6_{2}}$ m $= \frac{13}{8}$ m $= 1\frac{5}{8}$ m.

Hence, the length of each piece is $1\frac{5}{8}$ m.

EXAMPLE 6. *If the cost of $5\frac{2}{5}$ litres of milk is ₹ $236\frac{1}{4}$, find its cost per litre.*

Solution Cost of $\frac{27}{5}$ litres of milk = ₹ $\frac{945}{4}$

⇒ cost of 1 litre of milk = ₹ $\left(\frac{945}{4} \div \frac{27}{5}\right)$

= ₹ $\left(\frac{945^{35}}{4} \times \frac{5}{27_{1}}\right)$ $\left[\because \text{ reciprocal of } \frac{27}{5} \text{ is } \frac{5}{27}\right]$

= ₹ $\frac{175}{4}$ = ₹ $43\frac{3}{4}$.

Hence, the cost of milk per litre is ₹ $43\frac{3}{4}$.

EXAMPLE 7. *The cost of $5\frac{1}{4}$ kg of mangoes is ₹ 231. At what rate per kg are the mangoes being sold?*

Solution Cost of $\frac{21}{4}$ kg of mangoes = ₹ 231

⇒ cost of 1 kg of mangoes = ₹ $\left(231 \div \frac{21}{4}\right)$

= ₹ $\left(231^{11} \times \frac{4}{21_{1}}\right)$ $\left[\because \text{ reciprocal of } \frac{21}{4} \text{ is } \frac{4}{21}\right]$

= ₹ 44.

Hence, the mangoes are being sold at ₹ 44 per kg.

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EXAMPLE 8. *The product of two numbers is $15\frac{5}{6}$. If one of the numbers is $6\frac{2}{3}$, find the other.*

Solution Product of two numbers $= 15\frac{5}{6} = \frac{95}{6}$.

One of the numbers $= 6\frac{2}{3} = \frac{20}{3}$.

The other number $= \frac{95}{6} \div \frac{20}{3}$

$= \left(\frac{95}{6} \times \frac{3}{20}\right)$ $\left[\because \text{ reciprocal of } \frac{20}{3} \text{ is } \frac{3}{20}\right]$

$= \frac{95^{19} \times 3^{1}}{6_{2} \times 20_{4}} = \frac{19}{8} = 2\frac{3}{8}$.

Hence, the other number is $2\frac{3}{8}$.

EXAMPLE 9. *By what number should $6\frac{2}{9}$ be multiplied to get 40?*

Solution Product of two numbers = 40.

One of the numbers $= 6\frac{2}{9} = \frac{56}{9}$.

The other number $= \left(40 \div \frac{56}{9}\right) = \left(\frac{40}{1} \div \frac{56}{9}\right)$

$= \left(\frac{40}{1} \times \frac{9}{56}\right) = \frac{40^{5} \times 9}{1 \times 56_{7}} = \frac{45}{7} = 6\frac{3}{7}$.

Hence, the other number is $6\frac{3}{7}$.

EXERCISE 2C

1. Write down the reciprocal of:

   (i) $\frac{5}{8}$ (ii) 7 (iii) $\frac{1}{12}$ (iv) $12\frac{3}{5}$

2. Simplify:

   (i) $\frac{4}{7} \div \frac{9}{14}$ (ii) $\frac{7}{10} \div \frac{3}{5}$ (iii) $\frac{8}{9} \div 16$

   (iv) $9 \div \frac{1}{3}$ (v) $24 \div \frac{6}{7}$ (vi) $3\frac{3}{5} \div \frac{4}{5}$

   (vii) $3\frac{3}{7} \div \frac{8}{21}$ (viii) $5\frac{4}{7} \div 1\frac{3}{10}$ (ix) $15\frac{3}{7} \div 1\frac{23}{49}$

3. Divide:

   (i) $\frac{11}{24}$ by $\frac{7}{8}$ (ii) $6\frac{7}{8}$ by $\frac{11}{16}$ (iii) $5\frac{5}{9}$ by $3\frac{1}{3}$

   (iv) 32 by $1\frac{3}{5}$ (v) 45 by $1\frac{4}{5}$ (vi) 63 by $2\frac{1}{4}$

4. A rope of length $13\frac{1}{2}$ m has been divided into 9 pieces of the same length. What is the length of each piece?

5. 18 boxes of nails weigh equally and their total weight is $49\frac{1}{2}$ kg. How much does each box weigh?

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6. By selling oranges at the rate of ₹ $6\frac{3}{4}$ per orange, a man gets ₹ 378. How many oranges does he sell?

7. Mangoes are sold at ₹ $43\frac{1}{2}$ per kg. What is the weight of mangoes available for ₹ $326\frac{1}{4}$?

8. Vikas can cover a distance of $20\frac{2}{3}$ km in $7\frac{3}{4}$ hours on foot. How many km per hour does he walk?

9. Preeti bought $8\frac{1}{2}$ kg of sugar for ₹ $242\frac{1}{4}$. Find the price of sugar per kg.

10. If the cost of a notebook is ₹ $27\frac{3}{4}$, how many notebooks can be purchased for ₹ $249\frac{3}{4}$?

11. At a charity show the price of each ticket was ₹ $32\frac{1}{2}$. The total amount collected by a boy was ₹ $877\frac{1}{2}$. How many tickets were sold by him?

12. A group of students arranged a picnic. Each student contributed ₹ $261\frac{1}{2}$. The total contribution was ₹ $2876\frac{1}{2}$. How many students are there in the group?

13. 24 litres of milk was distributed equally among all the students of a hostel. If each student got $\frac{2}{5}$ litre of milk, how many students are there in the hostel?

14. A bucket contains $20\frac{1}{4}$ litres of water. A small jug has a capacity of $\frac{3}{4}$ litre. How many times the jug has to be filled with water from the bucket to get it emptied?

15. The product of two numbers is $15\frac{5}{6}$. If one of the numbers is $6\frac{1}{3}$, find the other.

16. By what number should $9\frac{4}{5}$ be multiplied to get 42?

17. By what number should $6\frac{2}{9}$ be divided to obtain $4\frac{2}{3}$?

EXERCISE 2D

OBJECTIVE QUESTIONS

Mark (✓) against the correct answer in each of the following:

1. Which of the following is a vulgar fraction?
   (a) $\frac{3}{10}$ (b) $\frac{13}{10}$ (c) $\frac{10}{3}$ (d) none of these

2. Which of the following is an improper fraction?
   (a) $\frac{7}{10}$ (b) $\frac{7}{9}$ (c) $\frac{9}{7}$ (d) none of these

3. Which of the following is a reducible fraction?
   (a) $\frac{105}{112}$ (b) $\frac{104}{121}$ (c) $\frac{77}{72}$ (d) $\frac{46}{63}$

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4. $\frac{2}{3}, \frac{4}{6}, \frac{6}{9}, \frac{8}{12}$ are

(a) like fractions
(b) irreducible fractions
(c) equivalent fractions
(d) none of these

5. Which of the following statements is true?

(a) $\frac{9}{16} = \frac{13}{24}$
(b) $\frac{9}{16} < \frac{13}{24}$
(c) $\frac{9}{16} > \frac{13}{24}$
(d) none of these

6. Reciprocal of $1\frac{3}{4}$ is

(a) $1\frac{4}{3}$
(b) $4\frac{1}{3}$
(c) $3\frac{1}{4}$
(d) none of these

7. $\left(\frac{3}{10} + \frac{8}{15}\right) = ?$

(a) $\frac{11}{10}$
(b) $\frac{11}{15}$
(c) $\frac{5}{6}$
(d) none of these

8. $\left(3\frac{1}{4} - 2\frac{1}{3}\right) = ?$

(a) $1\frac{1}{12}$
(b) $\frac{1}{12}$
(c) $1\frac{1}{11}$
(d) $\frac{11}{12}$

9. $36 \div \frac{1}{4} = ?$

(a) 9
(b) $\frac{1}{9}$
(c) $\frac{1}{144}$
(d) 144

10. By what number should $2\frac{3}{5}$ be multiplied to get $1\frac{6}{7}$?

(a) $1\frac{5}{7}$
(b) $\frac{5}{7}$
(c) $1\frac{1}{7}$
(d) $\frac{1}{7}$

11. By what number should $1\frac{1}{2}$ be divided to get $\frac{2}{3}$?

(a) $2\frac{2}{3}$
(b) $1\frac{2}{3}$
(c) $\frac{4}{9}$
(d) $2\frac{1}{4}$

12. $1\frac{3}{5} \div \frac{2}{3} = ?$

(a) $1\frac{1}{15}$
(b) $1\frac{9}{10}$
(c) $2\frac{2}{5}$
(d) none of these

13. $2\frac{1}{5} \div 1\frac{1}{5} = ?$

(a) 1
(b) 2
(c) $1\frac{1}{5}$
(d) $1\frac{5}{6}$

14. The reciprocal of $1\frac{2}{3}$ is

(a) $3\frac{1}{2}$
(b) $2\frac{1}{3}$
(c) $1\frac{1}{3}$
(d) $\frac{3}{5}$

15. Which one of the following is the correct statement?

(a) $\frac{2}{3} < \frac{3}{5} < \frac{14}{15}$
(b) $\frac{3}{5} < \frac{2}{3} < \frac{14}{15}$
(c) $\frac{14}{15} < \frac{3}{5} < \frac{2}{3}$
(d) none of these

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16. A car runs 16 km using 1 litre of petrol. How much distance will it cover in $2\frac{3}{4}$ litres of petrol?

(a) 24 km (b) 36 km (c) 44 km (d) $32\frac{3}{4}$ km

17. Lalit reads a book for $1\frac{3}{4}$ hours every day and reads the entire book in 6 days. How many hours does he take to read the entire book?

(a) $10\frac{1}{2}$ hours (b) $9\frac{1}{2}$ hours (c) $7\frac{1}{2}$ hours (d) $11\frac{1}{2}$ hours

Things to Remember

1. *The numbers of the form $\frac{a}{b}$, where a and b are natural numbers, are called fractions.*
2. *In $\frac{a}{b}$, we call a as numerator and b as denominator.*
3. *To get a fraction equivalent to a given fraction, we multiply (or divide) its numerator and denominator by the same nonzero number.*
4. *Fractions having same denominator are called like fractions. Otherwise, they are called unlike fractions.*
5. *In order to convert some given fractions into like fractions, we convert each one of them into an equivalent fraction having a denominator equal to the LCM of all the denominators of the given fractions.*
6. *A fraction whose numerator is less than its denominator is called a proper fraction. Otherwise, it is called an improper fraction.*
7. *A mixed fraction = A whole number + A fraction.*
8. *Let $\frac{a}{b}$ and $\frac{c}{d}$ be two given fractions.*
   Cross multiply them as $\frac{a}{b} \times \frac{c}{d}$.
   (i) *If $ad > bc$, then $\frac{a}{b} > \frac{c}{d}$.*
   (ii) *If $ad < bc$, then $\frac{a}{b} < \frac{c}{d}$.*
   (iii) *If $ad = bc$, then $\frac{a}{b} = \frac{c}{d}$.*
9. *Sum of like fractions = $\frac{\text{sum of their numerators}}{\text{common denominator}}$.*
10. *For adding unlike fractions, change them into equivalent like fractions and then add.*
11. *Difference of like fractions = $\frac{\text{difference of their numerators}}{\text{common denominator}}$.*
12. *For subtracting unlike fractions, change them into equivalent like fractions and then subtract.*
13. $\left(\frac{a}{b} \times \frac{c}{d}\right) = \frac{(a \times c)}{(b \times d)}$.
14. *Reciprocal of a nonzero fraction $\frac{a}{b}$ is $\frac{b}{a}$.*
15. $\left(\frac{a}{b} \div \frac{c}{d}\right) = \left(\frac{a}{b} \times \frac{d}{c}\right)$.

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TEST PAPER-2

A. 1. Define: (i) Fractions (ii) Vulgar fractions (iii) Improper fractions
Give two examples of each.

2. What should be added to $6\frac{3}{5}$ to get 15?

3. Simplify: $9\frac{5}{6}-4\frac{3}{8}+2\frac{7}{12}$.

4. Find: (i) $\frac{12}{25}$ of a litre (ii) $\frac{5}{8}$ of a kilogram (iii) $\frac{3}{5}$ of an hour

5. Milk is sold at ₹ $37\frac{3}{4}$ per litre. Find the cost of $6\frac{2}{5}$ litres of milk.

6. The cost of $5\frac{1}{4}$ kg of mangoes is ₹ 189. At what rate per kg are the mangoes being sold?

7. Simplify: (i) $1\frac{3}{4}\times2\frac{2}{5}\times3\frac{4}{7}$ (ii) $5\frac{5}{9}\div3\frac{1}{3}$

8. By what number should $6\frac{2}{9}$ be divided to obtain $4\frac{2}{3}$?

9. Each side of a square is $5\frac{2}{3}$ m long. Find its area.

B. ***Mark (✓) against the correct answer in each of the following:***

10. Which of the following is a vulgar fraction?
(a) $\frac{7}{10}$ (b) $\frac{19}{100}$ (c) $3\frac{3}{10}$ (d) $\frac{5}{8}$

11. Which of the following is an irreducible fraction?
(a) $\frac{105}{112}$ (b) $\frac{66}{77}$ (c) $\frac{46}{63}$ (d) $\frac{51}{85}$

12. Reciprocal of $1\frac{3}{5}$ is
(a) $1\frac{5}{3}$ (b) $5\frac{1}{3}$ (c) $3\frac{1}{5}$ (d) none of these

13. $1\frac{3}{5}\div\frac{2}{3}=?$
(a) $1\frac{9}{10}$ (b) $1\frac{1}{15}$ (c) $2\frac{2}{5}$ (d) none of these

14. Which of the following is correct?
(a) $\frac{2}{3}<\frac{3}{5}<\frac{11}{15}$ (b) $\frac{3}{5}<\frac{2}{3}<\frac{11}{15}$
(c) $\frac{11}{15}<\frac{3}{5}<\frac{2}{3}$ (d) $\frac{3}{5}<\frac{11}{15}<\frac{2}{3}$

15. By what number should $1\frac{3}{4}$ be divided to get $2\frac{1}{2}$?
(a) $\frac{3}{7}$ (b) $1\frac{2}{5}$ (c) $\frac{7}{10}$ (d) $1\frac{3}{7}$

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16. A car runs 9 km using 1 litre of petrol. How much distance will it cover in $3\frac{2}{3}$ litres of petrol?

(a) 36 km (b) 33 km (c) $2\frac{5}{11}$ km (d) 22 km

C. 17. ***Fill in the blanks.***

(i) Reciprocal of $8\frac{2}{5}$ is ....

(ii) $13\frac{1}{2} \div 8 =$ ......

(iii) $69\frac{3}{4} \div 7\frac{3}{4} =$ ......

(iv) $41\frac{2}{3} \times 18\frac{3}{5} =$......

(v) $\frac{84}{98}$ (in irreducible form) = ...

D. 18. ***Write 'T' for true and 'F' for false for each of the following:***

(i) $\frac{9}{16} < \frac{13}{24}$.

(ii) Among $\frac{2}{5}, \frac{16}{35}$ and $\frac{9}{14}$, the largest is $\frac{16}{35}$.

(iii) $\frac{11}{15} - \frac{9}{20} = \frac{17}{60}$.

(iv) $\frac{11}{25}$ of a litre = 440 mL.

(v) $16\frac{3}{4} \times 6\frac{2}{5} = 107\frac{3}{10}$.

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3 Decimals

In class VI we read about decimals, addition and subtraction of decimals, etc. We shall review these concepts in this chapter and take up multiplication and division of decimals.

Decimals *The numbers expressed in decimal forms are called decimals.*

EXAMPLES Each of the numbers 6.8, 16.73, 7.364, 0.053, etc., is a decimal.

A decimal has two parts, namely

(i) *whole-number part,* (ii) *decimal part.*

These parts are separated by a dot (·), called the decimal point.

The part on the left side of the decimal point is the whole-number part and that on its right side is the decimal part.

EXAMPLE In 73.62 we have, whole-number part = 73 and decimal part = .62.

Decimal places The number of digits contained in the decimal part of a decimal gives the number of decimal places.

EXAMPLES 5.74 has two decimal places and 8.536 has three decimal places.

Like decimals *Decimals having the same number of decimal places are called like decimals.*

EXAMPLES 6.73, 8.05, 19.68 are like decimals, each having two decimal places.

Unlike decimals *Decimals having different number of decimal places are called unlike decimals.*

EXAMPLES Clearly, 5.3, 8.64, 10.023 are unlike decimals.

An Important Result *Putting any number of zeros to the extreme right of the decimal part of a decimal does not change its value.*

Thus, 3.8 = 3.80 = 3.800, etc., 2.94 = 2.940 = 2.9400, etc.

EXAMPLE 1. *Arrange the digits of 374.568 in the place-value chart.*

Solution We may arrange the digits of the given number in the place-value chart as shown below:

Hundreds	Tens	Ones	Decimal point	Tenths	Hundredths	Thousandths
3	7	4	.	5	6	8

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Comparing Decimals

Suppose we have to compare two given decimals. We follow the following steps to do this.

Step 1. *Convert the given decimals into like decimals.*

Step 2. *First compare the whole-number part.*
The decimal with the greater whole-number part is greater.

Step 3. *If the whole-number parts are equal, compare the tenths digits.*
The decimal with the bigger digit in the tenths place is greater.

Step 4. *If the tenths digits are also equal, compare the hundredths digits, and so on.*

EXAMPLE 2. *Write the following decimals in ascending order:*
5.74, 6.03, 0.8, 0.658 *and* 7.2.

Solution Converting the given decimals into like decimals, we get them as:
5.740, 6.030, 0.800, 0.658 and 7.200.
Clearly, $0.658 < 0.800 < 5.740 < 6.030 < 7.200$.
Hence, the given decimals in ascending order are:
0.658, 0.8, 5.74, 6.03, 7.2.

METHOD OF CONVERTING A DECIMAL INTO A FRACTION

Step 1. *Write the given decimal without the decimal point as the numerator of the fraction.*

Step 2. *In the denominator, write 1 followed by as many zeros as there are decimal places in the given decimal.*

Step 3. *Reduce the above fraction to the simplest form.*

EXAMPLE 3. *Convert each of the following decimals into a fraction in its simplest form:*
(i) .5 *(ii)* .24 *(iii)* .08 *(iv)* .225

Solution We have:

(i) $.5 = \frac{\cancel{5}^1}{\cancel{10}_2} = \frac{1}{2}$. (ii) $.24 = \frac{\cancel{24}^6}{\cancel{100}_{25}} = \frac{6}{25}$.

(iii) $.08 = \frac{\cancel{8}^2}{\cancel{100}_{25}} = \frac{2}{25}$. (iv) $.225 = \frac{\cancel{225}^9}{\cancel{1000}_{40}} = \frac{9}{40}$.

CONVERTING A FRACTION INTO A DECIMAL

Step 1. *Divide the numerator by the denominator till a nonzero remainder is obtained.*

Step 2. *Put a decimal point in the dividend as well as in the quotient.*

Step 3. *Put a zero on the right of the decimal point in the dividend as well as on the right of the remainder.*

Step 4. *Divide again just as we do in whole numbers.*

Step 5. *Repeat steps 3 and 4, till the remainder is zero.*

EXAMPLE 4. *Convert each of the following into a decimal fraction:*

(i) $\frac{27}{4}$ *(ii)* $2\frac{5}{8}$

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Solution On dividing, we get:

(i) $\frac{27}{4}$

$$\begin{array}{r} 6.75 \\ 4\overline{)27.00} \\ -24 \\ \hline 30 \\ -28 \\ \hline 20 \\ -20 \\ \hline \times \end{array}$$

$\therefore \frac{27}{4} = 6.75.$

(ii) $2\frac{5}{8} = \frac{21}{8}$

$$\begin{array}{r} 2.625 \\ 8\overline{)21.000} \\ -16 \\ \hline 50 \\ -48 \\ \hline 20 \\ -16 \\ \hline 40 \\ -40 \\ \hline \times \end{array}$$

$\therefore 2\frac{5}{8} = \frac{21}{8} = 2.625.$

EXAMPLE 5. *Express as rupees using decimals:*

(i) 536 *paise* *(ii)* 65 *rupees* 78 *paise* *(iii)* 6 *rupees* 6 *paise*

(iv) 28 *paise* *(v)* 5 *paise*

Solution We have:

(i) 536 paise = ₹ $\frac{536}{100}$ = ₹ 5.36.

(ii) 65 rupees 78 paise = ₹ 65.78.

(iii) 6 rupees 6 paise = $(6\times100+6)$ paise = 606 paise = ₹ $\frac{606}{100}$ = ₹ 6.06.

(iv) 28 paise = ₹ $\frac{28}{100}$ = ₹ 0.28.

(v) 5 paise = ₹ $\frac{5}{100}$ = ₹ 0.05.

EXAMPLE 6. *Express* 5 cm *in metre and kilometre.*

Solution 5 cm = $\frac{5}{100}$ m = 0.05 m

= $\frac{0.05}{1000}$ km = 0.00005 km.

$\therefore$ 5 cm = 0.05 m = 0.00005 km.

EXAMPLE 7. *Express in* kg *using decimals:*

(i) 60 g *(ii)* 7380 g *(iii)* 6 kg 8 g

Solution We have:

(i) 60 g = $\frac{60}{1000}$ kg = $\frac{6}{100}$ kg = 0.06 kg.

(ii) 7380 g = $\frac{7380}{1000}$ kg = 7.380 kg.

(iii) 6 kg 8 g = (6×1000) g + 8 g = (6008) g

= $\frac{6008}{1000}$ kg = 6.008 kg.

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EXERCISE 3A

1. Convert each of the following into a fraction in its simplest form:
   (i) .8 (ii) .75 (iii) .06 (iv) .285
2. Convert each of the following as a mixed fraction:
   (i) 5.6 (ii) 12.25 (iii) 6.004 (iv) 4.625
3. Convert each of the following into a decimal:
   (i) $\frac{47}{10}$ (ii) $\frac{156}{100}$ (iii) $\frac{2516}{100}$ (iv) $\frac{3524}{1000}$
   (v) $\frac{25}{8}$ (vi) $3\frac{2}{5}$ (vii) $2\frac{2}{25}$ (viii) $\frac{17}{20}$
4. Convert each of the following into like decimals:
   (i) 6.5, 16.03, 0.274, 119.4 (ii) 3.5, 0.67, 15.6, 4
5. Fill in each of the place holders with the correct symbol > or <.
   (i) 78.23 ☐ 69.85 (ii) 3.406 ☐ 3.46
   (iii) 5.68 ☐ 5.86 (iv) 14.05 ☐ 14.005
   (v) 1.85 ☐ 1.805 (vi) 0.98 ☐ 1.07
6. Arrange the following decimals in ascending order:
   (i) 4.6, 7.4, 4.58, 7.32, 4.06 (ii) 0.5, 5.5, 5.05, 0.05, 5.55
   (iii) 6.84, 6.48, 6.8, 6.4, 6.08 (iv) 2.2, 2.202, 2.02, 22.2, 2.002
7. Arrange the following decimals in descending order:
   (i) 7.4, 8.34, 74.4, 7.44, 0.74 (ii) 2.6, 2.26, 2.06, 2.007, 2.3
8. Express 45 mm in cm, m and km.
9. Express as rupees using decimals:
   (i) 8 paise (ii) 9 rupees 75 paise (iii) 8 rupees 5 paise
10. Express in km using decimals:
   (i) 65 m (ii) 284 m (iii) 3 km 5 m

ADDITION AND SUBTRACTION OF DECIMALS

ADDITION OF DECIMALS

METHOD:

Step 1. *Convert the given decimals into like decimals.*

Step 2. *Write the addends one under the other in column form, keeping the decimal points of all the addends in the same column and the digits of the same place in the same column.*

Step 3. *Add as in the case of whole numbers.*

Step 4. *In the sum, put the decimal point directly under decimal points in the addends.*

EXAMPLE 1. *Add* 36.4, 273.06, 9.397 *and* 68.

Solution Converting the given decimals into like decimals, we get:
36.400, 273.060, 9.397 and 68.000.

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Writing these decimals in column form and adding, we get:

$$\begin{array}{r} 36.400 \\ 273.060 \\ 9.397 \\ 68.000 \\ \hline 386.857 \\ \hline \end{array}$$

Hence, the sum of the given decimals is 386.857.

SUBTRACTION OF DECIMALS

METHOD:

Step 1. *Convert the given decimals into like decimals.*

Step 2. *Write the smaller number under the larger one in column form in such a way that the decimal points of both the numbers are in the same column and the digits of the same place lie in the same column.*

Step 3. *Subtract as we do in case of whole numbers.*

Step 4. *In the difference, put the decimal point directly under the decimal points of the given numbers.*

EXAMPLE 2. *Subtract* 47.56 *from* 83.2.

Solution Converting the given decimals into like decimals, we get 47.56 and 83.20. Writing them in column form with the larger one at the top and subtracting, we get:

$$\begin{array}{r} 83.20 \\ -47.56 \\ \hline 35.64 \\ \hline \end{array}$$

Hence, $(83.20 - 47.56) = 35.64$.

EXAMPLE 3. *Simplify:* $63.7 - 28.89 + 76.4 - 37.66$.

Solution Converting the given decimals into like decimals, we have:

$63.7 - 28.89 + 76.4 - 37.66$
$= 63.70 - 28.89 + 76.40 - 37.66$
$= (63.70 + 76.40) - (28.89 + 37.66)$
$= (140.10 - 66.55)$
$= 73.55.$

$$\begin{array}{r} 63.70 \\ +76.40 \\ \hline 140.10 \end{array} \qquad \begin{array}{r} 28.89 \\ +37.66 \\ \hline 66.55 \end{array}$$

$$\begin{array}{r} 140.10 \\ -66.55 \\ \hline 73.55 \\ \hline \end{array}$$

EXAMPLE 4. *How much less is* 28.8 km *than* 42.3 km?

Solution Required difference

$= (42.3 - 28.8)$ km
$= 13.5$ km.

$$\begin{array}{r} 42.3 \\ -28.8 \\ \hline 13.5 \\ \hline \end{array}$$

EXAMPLE 5. *Shayama bought* 4 kg 350 g *potato,* 3 kg 80 g *tomato and some onion. If the total weight of the three vegetables is* 10 kg 200 g, *what is the weight of onion?*

Solution Total weight of all the vegetables = 10 kg 200 g = 10.200 kg.

Weight of potato = 4 kg 350 g = 4.350 kg.

Weight of tomato = 3 kg 80 g = 3.080 kg.

Weight of onion = [10.200 − (4.350 + 3.080)] kg
= (10.200 − 7.430) kg = 2.770 kg.

Hence, the weight of onion is 2 kg 770 g.

$$\begin{array}{r} 4.350 \\ +3.080 \\ \hline 7.430 \\ \hline \end{array} \qquad \begin{array}{r} 10.200 \\ -7.430 \\ \hline 2.770 \\ \hline \end{array}$$

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EXERCISE 3B

Add:

1. 16, 8.7, 0.94, 6.8 and 7.77
2. 18.6, 206.37, 8.008, 26.4 and 6.9
3. 63.5, 9.7, 0.8, 26.66 and 12.17
4. 17.4, 86.39, 9.435, 8.8 and 0.06
5. 26.9, 19.74, 231.769 and 0.048
6. 23.8, 8.94, 0.078 and 214.6
7. 6.606, 66.6, 666, 0.066, 0.66
8. 9.09, 0.909, 99.9, 9.99, 0.099

Subtract:

9. 14.79 from 72.43
10. 36.74 from 52.6
11. 13.876 from 22
12. 15.079 from 24.16
13. 0.68 from 1.007
14. 0.4678 from 5.05
15. 2.5307 from 8
16. 6.732 from 9.001
17. Take out 5.746 from 9.1.
18. What is to be added to 63.58 to get 92?
19. What is to be subtracted from 8.1 to get 0.813?
20. By how much should 32.67 be increased to get 60.1?
21. By how much should 74.3 be decreased to get 26.87?
22. Rohit purchased a notebook for ₹ 23.75, a pencil for ₹ 2.85 and a pen for ₹ 15.90. He gave a 50-rupee note to the shopkeeper. What amount did he get back?

MULTIPLICATION OF DECIMALS

MULTIPLICATION OF A DECIMAL BY 10, 100, 1000, etc.

Rules: *(i) On multiplying a decimal by* 10, *the decimal point is shifted to the right by one place.*

(ii) On multiplying a decimal by 100, *the decimal point is shifted to the right by two places.*

(iii) On multiplying a decimal by 1000, *the decimal point is shifted to the right by three places, and so on.*

EXAMPLE 1. *Find the product:*
(i) 67.24×10 *(ii)* 4.956×100 *(iii)* 2.3748×1000

Solution We have:
(i) $67.24 \times 10 = 672.4$ [shifting decimal point to the right by 1 place]
(ii) $4.956 \times 100 = 495.6$ [shifting decimal point to the right by 2 places]
(iii) $2.3748 \times 1000 = 2374.8$ [shifting decimal point to the right by 3 places]

EXAMPLE 2. *Multiply* 35.6 *by* 1000.

Solution On multiplying 35.6 by 1000, the decimal point will be shifted to the right by 3 places.

So, we write, 35.6 = 35.600.

$\therefore$ $35.6 \times 1000 = 35.600 \times 1000 = 35600.$

Hence, $35.6 \times 1000 = 35600.$

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MULTIPLICATION OF A DECIMAL BY A WHOLE NUMBER

METHOD:

Step 1. *Multiply the decimal without the decimal point by the given whole number.*

Step 2. *Mark the decimal point in the product to have as many places of decimal as are there are in the given decimal.*

EXAMPLE 3. *Find the product:*

(i) 5.43×15 (ii) 0.327×12 (iii) 0.065×9

Solution We have:

(i) $543 \times 15 = 8145$.

$\therefore$ $5.43 \times 15 = 81.45$. (2 places of decimal)

(ii) $327 \times 12 = 3924$.

$\therefore$ $0.327 \times 12 = 3.924$. (3 places of decimal)

(iii) $65 \times 9 = 585$.

$\therefore$ $0.065 \times 9 = 0.585$. (3 places of decimal)

MULTIPLICATION OF A DECIMAL BY A DECIMAL

METHOD:

Step 1. *Multiply the two decimals without the decimal point just like whole numbers.*

Step 2. *Mark the decimal point in the product in such a way that the number of decimal places in the product is equal to the sum of the decimal places in the given decimals.*

EXAMPLE 4. *Multiply* 73.68 *by* 5.4.

Solution First we multiply 7368 by 54.

```
   7368
    ×54
  29472
 368400
 397872
```

$\therefore$ $7368 \times 54 = 397872$.

Sum of decimal places in the given decimals = $(2+1) = 3$.

So, the product must contain 3 places of decimal.

$\therefore$ $73.68 \times 5.4 = 397.872$. (3 places of decimal)

EXAMPLE 5. *Multiply* 0.089 *by* 0.76.

Solution First we multiply 89 by 76.

```
   89
  ×76
  534
 6230
 6764
```

$\therefore$ $89 \times 76 = 6764$.

Sum of decimal places in the given decimals = $(3 + 2) = 5$.

So, the product must contain 5 places of decimal.

$\therefore$ $0.089 \times 0.76 = 0.06764$. (5 places of decimal)

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EXAMPLE 6. *Multiply 0.0235 by 0.0327.*

Solution First we multiply 235 by 327.

$$\begin{array}{r} 235 \\ \times 327 \\ \hline 1645 \\ 4700 \\ 70500 \\ \hline 76845 \\ \hline \end{array}$$

$\therefore$ $235 \times 327 = 76845$.

Sum of decimal places in the given decimals = (4 + 4) =8.

So, the product must contain 8 places of decimal.

$\therefore$ $0.0235 \times 0.0327 = 0.00076845$.

EXAMPLE 7. *Find the product* $0.47 \times 5.3 \times 0.06$.

Solution First we find the product $47 \times 53 \times 6$.

Now, $47 \times 53 \times 6 = 2491 \times 6 = 14946$.

Sum of decimal places in the given decimals = (2 + 1 + 2) = 5.

So, the product must contain 5 places of decimal.

$\therefore$ $0.47 \times 5.3 \times 0.06 = 0.14946$.

$$\begin{array}{r} 47 \\ \times 53 \\ \hline 141 \\ 2350 \\ \hline 2491 \\ \times 6 \\ \hline 14946 \\ \hline \end{array}$$

EXAMPLE 8. *If the cost of a pen is* ₹ 28.50, *find the cost of* 48 *such pens.*

Solution Cost of 1 pen = ₹ 28.50.

Cost of 48 pens = ₹ (28.50×48)

= ₹ 1368.00

= ₹ 1368.

Hence, the cost of 48 pens is ₹ 1368.

$$\begin{array}{r} 2850 \\ \times 48 \\ \hline 22800 \\ 114000 \\ \hline 136800 \\ \hline \end{array}$$

EXAMPLE 9. *The cost of* 1 *metre of ribbon is* ₹ 35.80. *What will be the cost of* 9.8 *metres of ribbon?*

Solution Cost of 1 m of ribbon = ₹ 35.80.

Cost of 9.8 m of ribbon = ₹ (35.80×9.8)

= ₹ 350.840

= ₹ 350.84.

Hence, the cost of 9.8 m of ribbon is ₹ 350.84.

$$\begin{array}{r} 3580 \\ \times 98 \\ \hline 28640 \\ 322200 \\ \hline 350840 \\ \hline \end{array}$$

EXAMPLE 10. 1 kg *of milk has* 0.264 kg *of fat. How much fat is there in* 12.5 kg *of milk?*

Solution Quantity of fat in 1 kg of milk = 0.264 kg.

Quantity of fat in 12.5 kg of milk = (0.264×12.5) kg

= 3.3000 kg

= 3.3 kg.

Hence, the quantity of fat in 12.5 kg of milk is 3.3 kg.

$$\begin{array}{r} 264 \\ \times 125 \\ \hline 1320 \\ 5280 \\ 26400 \\ \hline 33000 \\ \hline \end{array}$$

EXERCISE 3C

1. Find the product:

(i) 73.92×10 (ii) 7.54×10 (iii) 84.003×10

(iv) 0.83×10 (v) 0.7×10 (vi) 0.032×10

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2. Find the product:

(i) 2.397×100 (ii) 6.83×100 (iii) 2.9×100

(iv) 0.08×100 (v) 0.6×100 (vi) 0.003×100

3. Find the product:

(i) 6.7314×1000 (ii) 0.182×1000 (iii) 0.076×1000

(iv) 6.25×1000 (v) 4.8×1000 (vi) 0.06×1000

4. Find the product:

(i) 5.4×16 (ii) 3.65×19 (iii) 0.854×12

(iv) 36.73×48 (v) 4.125×86 (vi) 104.06×75

(vii) 6.032×124 (viii) 0.0146×69 (ix) 0.00125×327

5. Find the product:

(i) 7.6×2.4 (ii) 3.45×6.3 (iii) 0.54×0.27

(iv) 0.568×4.9 (v) 6.54×0.09 (vi) 3.87×1.25

(vii) 0.06×0.38 (viii) 0.623×0.75 (ix) 0.014×0.46

(x) 54.5×1.76 (xi) 0.045×2.4 (xii) 1.245×6.4

6. Find the product:

(i) $13 \times 1.3 \times 0.13$ (ii) $2.4 \times 1.5 \times 2.5$ (iii) $0.8 \times 3.5 \times 0.05$

(iv) $0.2 \times 0.02 \times 0.002$ (v) $11.1 \times 1.1 \times 0.11$ (vi) $2.1 \times 0.21 \times 0.021$

7. Evaluate:

(i) $(1.2)^2$ (ii) $(0.7)^2$ (iii) $(0.04)^2$ (iv) $(0.11)^2$

8. Evaluate:

(i) $(0.3)^3$ (ii) $(0.05)^3$ (iii) $(1.5)^3$

9. A bus can cover 62.5 km in one hour. How much distance can it cover in 18 hours?
10. A tin of oil weighs 16.8 kg. What is the weight of 45 such tins?
11. A bag of wheat weighs 97.8 kg. How much wheat is contained in 500 such bags?
12. Find the weight of 16 bags of sugar, each weighing 48.450 kg.
13. A small bottle holds 0.845 kg of sauce. How much sauce will be there in 72 such bottles?
14. A bottle holds 925 g of jam. How many kg of jam will be there in 25 such bottles?
15. If one drum can hold 16.850 litres of oil, how many litres can 48 such drums hold?
16. 1 kg of rice costs ₹ 56.80. What is the cost of 16.25 kg of rice?
17. 1 metre of cloth costs ₹ 108.50. What is the cost of 18.5 metres of this cloth?
18. A car can cover a distance of 8.6 km on one litre of petrol. How far can it go on 36.5 litres of petrol?
19. A taxi driver charges ₹ 9.80 per km. How much will he charge for a journey of 106.5 km?

DIVISION OF DECIMALS

DIVIDING A DECIMAL BY 10, 100, 1000, etc.

Rules: *(i) On dividing a decimal by 10, the decimal point is shifted to the left by one place.*

(ii) On dividing a decimal by 100, the decimal point is shifted to the left by two places.

(iii) On dividing a decimal by 1000, the decimal point is shifted to the left by three places, and so on.

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EXAMPLE 1. *Divide:*
(i) 16.8 *by* 10 *(ii)* 236.4 *by* 100 *(iii)* 3709.6 *by* 1000

Solution We have:

(i) $16.8 \div 10 = \frac{16.8}{10} = 1.68$ [shifting decimal point to the left by 1 place]

(ii) $236.4 \div 100 = \frac{236.4}{100} = 2.364$ [shifting decimal point to the left by 2 places]

(iii) $3709.6 \div 1000 = \frac{3709.6}{1000} = 3.7096$ [shifting decimal point to the left by 3 places]

EXAMPLE 2. *Divide:*
(i) 0.46 *by* 10 *(ii)* 2.34 *by* 100 *(iii)* 6.28 *by* 1000

Solution We have:

(i) $0.46 \div 10 = \frac{0.46}{10} = 0.046$ [shifting decimal point to the left by 1 place]

(ii) $2.34 \div 100 = \frac{2.34}{100} = 0.0234$ [shifting decimal point to the left by 2 places]

(iii) $6.28 \div 1000 = \frac{6.28}{1000} = 0.00628$ [shifting decimal point to the left by 3 places]

DIVIDING A DECIMAL BY A WHOLE NUMBER

METHOD:

Step 1. *Perform the division by considering the dividend a whole number.*

Step 2. *When the division of whole-number part of the dividend is complete, put the decimal point in the quotient and proceed with the division as in case of whole numbers.*

EXAMPLE 3. *Divide* 39.168 *by* 12.

Solution We have:

```
12)39.168(3.264
  -36
    31
   -24
     76
    -72
     48
    -48
      0
```

$\therefore$ $39.168 \div 12 = 3.264$.

EXAMPLE 4. *Divide* 0.567 *by* 9.

Solution We have:

```
9)0.567(0.063
 -0
   56
  -54
    27
   -27
     0
```

$\therefore$ $0.567 \div 9 = 0.063$.

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REMARK Sometimes on dividing a decimal by a whole number, the last remainder obtained is nonzero. In such cases insert as many zeros on the right of decimal part of the dividend as necessary to make the last remainder zero.

EXAMPLE 5. *Divide* 2.32 *by* 16.

Solution We have:

```
     0.145
16)2.320    ← one zero annexed
  -0
   23
  -16
    72
   -64
     80
    -80
      0
```

$\therefore\ 2.32 \div 16 = 0.145.$

EXAMPLE 6. *Divide* 48.38 *by* 8.

Solution We have:

```
    6.0475
8)48.3800    ← two zeros annexed
 -48
    38
   -32
     60
    -56
      40
     -40
       0
```

$\therefore\ 48.38 \div 8 = 6.0475.$

EXAMPLE 7. *Divide* 202.4 *by* 40.

Solution We have:

$$\frac{202.4}{40} = \frac{202.4}{4 \times 10} = \frac{202.4}{4} \times \frac{1}{10} = \frac{50.6}{10} = 5.06.$$

EXAMPLE 8. *Find the quotient:*

(*i*) $0.018 \div 0.6$ (*ii*) $0.0018 \div 0.09$ (*iii*) $0.196 \div 1.4$

Solution We have:

(i) $\frac{0.018}{0.6} = \frac{0.018 \times 10}{0.6 \times 10} = \frac{0.18}{6}$

```
6)0.18(0.03
 -0
   18
  -18
    0
```

$\therefore\ \frac{0.018}{0.6} = \frac{0.18}{6} = 0.03.$

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(ii) $\frac{0.0018}{0.09} = \frac{0.0018 \times 100}{0.09 \times 100} = \frac{0.18}{9}$

$$
\begin{array}{l}
9\overline{)0.18}(0.02 \\
\underline{-0} \\
\quad 18 \\
\quad \underline{-18} \\
\quad\ 0
\end{array}
$$

$\therefore \frac{0.0018}{0.09} = \frac{0.18}{9} = 0.02.$

(iii) $\frac{0.196}{1.4} = \frac{0.196 \times 10}{1.4 \times 10} = \frac{1.96}{14}$

$$
\begin{array}{l}
14\overline{)1.96}(0.14 \\
\underline{-0} \\
\ 19 \\
\underline{-14} \\
\quad 56 \\
\quad \underline{-56} \\
\quad\ 0
\end{array}
$$

$\therefore \frac{0.196}{1.4} = \frac{1.96}{14} = 0.14.$

EXAMPLE 9. *A bowler took 15 wickets for 321 runs. What is his average score per wicket?*

Solution Total score = 321 runs.

Total number of wickets = 15.

Average score per wicket = $\frac{321}{15}$ runs = 21.4 runs.

Hence, the average score is 21.4 runs per wicket.

EXAMPLE 10. *A car covers a distance of 108.9 km in 1.8 hours. What is the average speed of the car?*

Solution Total distance covered = 108.9 km.

Total time taken = 1.8 hours.

Average speed of the car = $\frac{\text{distance}}{\text{time taken}}$

$= \frac{108.9}{1.8}$ km/h $= \frac{1089}{18}$ km/h

$= \frac{121}{2}$ km/h = 60.5 km/h.

Hence, the average speed of the car is 60.5 km/h.

EXAMPLE 11. *The cost of 24 toys of the same kind is ₹ 783.60. Find the cost of each toy.*

Solution Cost of 24 toys = ₹ 783.60.

Cost of 1 toy = ₹$\left(\frac{783.60}{24}\right)$

= ₹ 32.65.

Hence, the cost of each toy is ₹ 32.65.

$$
\begin{array}{l}
24\overline{)783.60}(32.65 \\
\underline{-72} \\
\quad 63 \\
\underline{-48} \\
\quad 156 \\
\underline{-144} \\
\quad\ 120 \\
\quad \underline{-120} \\
\qquad 0
\end{array}
$$

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EXAMPLE 12. *The total weight of some bags of cement is 1743 kg. If each bag weighs 49.8 kg, how many bags are there?*

Solution Total weight of all the bags = 1743 kg.

Weight of each bag = 49.8 kg.

$$\text{Number of bags} = \frac{\text{total weight}}{\text{weight of each bag}}$$

$$= \frac{1743}{49.8} = \left(\frac{1743}{49.8} \times \frac{10}{10}\right)$$

$$= \frac{17430}{498} = 35.$$

Hence, the required number of bags = 35.

```
498)17430(35
   -1494
     2490
    -2490
        0
```

EXAMPLE 13. *Mr Thukral distributed ₹ 1840 equally among NCC cadets for refreshment. If each cadet received ₹ 28.75, how many cadets were there?*

Solution Total amount distributed = ₹ 1840.

Amount received by each cadet = ₹ 28.75.

$$\text{Number of cadets} = \frac{\text{total amount}}{\text{amount received by each}}$$

$$= \frac{1840}{28.75} = \frac{1840}{28.75} \times \frac{100}{100}$$

$$= \frac{184000}{2875} = 64.$$

Hence, there were 64 cadets in all.

```
            64
2875)184000(
    -17250
      11500
     -11500
          0
```

EXAMPLE 14. *Mrs Bose bought 15.5 litres of refined oil for ₹ 1122.20. Find its cost per litre.*

Solution Cost of 15.5 litres of refined oil = ₹ 1122.20.

$$\text{Cost of 1 litre of refined oil} = ₹\left(\frac{1122.20}{15.5}\right)$$

$$= ₹\left(\frac{1122.20}{15.5} \times \frac{10}{10}\right)$$

$$= ₹\left(\frac{11222}{155}\right) = ₹\ 72.40.$$

Hence, the cost of refined oil is ₹ 72.40 per litre.

```
         72.40
155)11222.00(
   -1085
      372
     -310
       620
      -620
        00
       -00
         0
```

EXAMPLE 15. *The product of two decimals is 1.5008. If one of them is 0.56, find the other.*

Solution Product of given decimals = 1.5008.

One decimal = 0.56.

The other decimal = 1.5008 ÷ 0.56

$$= \left(\frac{1.5008}{0.56} \times \frac{100}{100}\right)$$

$$= \frac{150.08}{56} = 2.68.$$

Hence, the other decimal is 2.68.

```
56)150.08(2.68
  -112
    380
   -336
     448
    -448
       0
```

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EXAMPLE 16. *Each side of a polygon is 2.9 cm in length and its perimeter is 17.4 cm. How many sides does the polygon have?*

Solution Let the number of sides of the polygon be n.

Length of each side of the polygon = 2.9 cm.

$\therefore$ perimeter of the polygon = $(2.9 \times n)$ cm.

But, its perimeter = 17.4 cm (given).

$\therefore\ 2.9 \times n = 17.4 \Rightarrow n = \frac{17.4}{2.9} = \frac{174}{29} = 6.$

Hence, the given polygon has 6 sides.

EXAMPLE 17. *Find the average of 4.2, 7.4 and 8.8.*

Solution Average of the given numbers $= \frac{(4.2 + 7.4 + 8.8)}{3}$

$= \frac{20.4}{3} = 6.8.$

Hence, the average of the given numbers is 6.8.

EXERCISE 3D

1. Divide:
(i) 131.6 by 10 (ii) 32.56 by 10 (iii) 4.38 by 10
(iv) 0.34 by 10 (v) 0.08 by 10 (vi) 0.062 by 10

2. Divide:
(i) 137.2 by 100 (ii) 23.4 by 100 (iii) 4.7 by 100
(iv) 0.3 by 100 (v) 0.58 by 100 (vi) 0.02 by 100

3. Divide:
(i) 1286.5 by 1000 (ii) 354.16 by 1000 (iii) 38.9 by 1000
(iv) 4.6 by 1000 (v) 0.8 by 1000 (vi) 2 by 1000

4. Divide:
(i) 12 by 8 (ii) 63 by 15 (iii) 47 by 20
(iv) 101 by 25 (v) 31 by 40 (vi) 11 by 16

5. Divide:
(i) 43.2 by 6 (ii) 60.48 by 12 (iii) 117.6 by 21
(iv) 217.44 by 18 (v) 2.575 by 25 (vi) 6.08 by 8
(vii) 0.765 by 9 (viii) 0.768 by 16 (ix) 0.175 by 25
(x) 0.3322 by 11 (xi) 2.13 by 15 (xii) 6.54 by 12
(xiii) 5.52 by 16 (xiv) 1.001 by 14 (xv) 0.477 by 18

6. Divide:
(i) $16.46 \div 20$ (ii) $403.8 \div 30$ (iii) $19.2 \div 80$
(iv) $156.8 \div 200$ (v) $12.8 \div 500$ (vi) $18.08 \div 400$

7. Divide:
(i) 3.28 by 0.8 (ii) 0.288 by 0.9 (iii) 25.395 by 1.5
(iv) 2.0484 by 0.18 (v) 0.228 by 0.38 (vi) 0.8085 by 0.35
(vii) 21.976 by 1.64 (viii) 11.04 by 1.6 (ix) 6.612 by 11.6
(x) 0.076 by 0.19 (xi) 148 by 0.074 (xii) 16.578 by 5.4
(xiii) 28 by 0.56 (xiv) 204 by 0.17 (xv) 3 by 80

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8. The total cost of 24 chairs is ₹ 9255.60. Find the cost of each chair.
9. 1.8 m of cloth is required for a shirt. How many such shirts can be made from a piece of cloth 45 m long?
10. A car covers a distance of 22.8 km in 2.4 litres of petrol. How much distance will it cover in 1 litre of petrol?
11. A tin holds 16.5 litres of oil. How many such tins will be required to hold 478.5 litres of oil?
12. The weight of 37 bags of sugar is 3644.5 kg. If all the bags weigh equally, what is the weight of each bag?
13. If 69 buckets of equal capacity can be filled with 586.5 litres of water, what is the capacity of each bucket?
14. Monica cuts 46 m of cloth into pieces of 1.15 m each. How many pieces does she get?
15. Mr Soni bought some bags of cement, each weighing 49.8 kg. If the total weight of all the bags is 1792.8 kg, how many bags did he buy?
16. How many pieces of plywood, each 0.35 cm thick, are required to make a pile 1.89 m high?
Hint. 1.89 m = (1.89×100) cm = 189 cm.
17. The product of two decimals is 261.36. If one of them is 17.6, find the other.

EXERCISE 3E

OBJECTIVE QUESTIONS

Mark (✓) against the correct answer in each of the following:

1. .06 = ?
(a) $\frac{3}{5}$ (b) $\frac{3}{50}$ (c) $\frac{3}{500}$ (d) none of these

2. 1.04 = ?
(a) $1\frac{1}{5}$ (b) $1\frac{2}{5}$ (c) $1\frac{1}{25}$ (d) none of these

3. $2\frac{2}{25} = ?$
(a) 2.8 (b) 2.08 (c) 2.008 (d) none of these

4. 6 cm = ?
(a) 0.006 km (b) 0.0006 km (c) 0.00006 km (d) none of these

5. 70 g = ?
(a) 0.7 kg (b) 0.07 kg (c) 0.007 kg (d) none of these

6. 5 kg 6 g = ?
(a) 5.0006 kg (b) 5.06 kg (c) 5.006 kg (d) 5.6 kg

7. 2 km 5 m = ?
(a) 2.5 km (b) 2.05 km (c) 2.005 km (d) 2.0005 km

8. (1.007 − 0.7) = ?
(a) 1 (b) 0.37 (c) 0.307 (d) none of these

9. What should be subtracted from .1 to get .03?
(a) .7 (b) .07 (c) .007 (d) none of these

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10. What should be added to 3.07 to get 3.5?
(a) .57 (b) .34 (c) .43 (d) .02

11. $0.23 \times 0.3 = ?$
(a) 0.69 (b) 6.9 (c) 0.069 (d) none of these

12. $0.02 \times 30 = ?$
(a) 6 (b) 0.6 (c) 0.06 (d) none of these

13. $0.25 \times 0.8 = ?$
(a) 0.02 (b) 0.2 (c) 0.002 (d) 2

14. $0.4 \times 0.4 \times 0.4 = ?$
(a) 6.4 (b) .64 (c) .064 (d) none of these

15. $1.1 \times .1 \times .01 = ?$
(a) .011 (b) .0011 (c) .11 (d) none of these

16. $2.08 \div (.16) = ?$
(a) 13 (b) .13 (c) 1.3 (d) none of these

17. $1.02 \div 6 = ?$
(a) 1.7 (b) 0.17 (c) 0.017 (d) none of these

18. $30.94 \div 0.7 = ?$
(a) 44.2 (b) 4.42 (c) 442 (d) 0.442

19. $2.73 \div 1.3 = ?$
(a) 21 (b) 2.1 (c) 0.21 (d) none of these

20. $89.1 \div 2.2 = ?$
(a) 40.5 (b) 4.05 (c) 41 (d) 41.5

21. $0.5 \times 0.05 = ?$
(a) 0.25 (b) 2.5 (c) 0.025 (d) none of these

Things to Remember

1. *The fractions in which the denominators are 10, 100, 1000, etc., are known as decimal fractions.*
2. *Numbers written in decimal form are called decimals.*
3. *A decimal has two parts, namely, the whole-number part and the decimal part.*
4. *The number of digits contained in the decimal part of a decimal is called the number of its decimal places.*
5. *Decimals having the same number of decimal places are called like decimals, otherwise they are known as unlike decimals.*
6. *We have* $0.1 = 0.10 = 0.100$, *etc.,* $0.2 = 0.20 = 0.200$, *etc., and so on.*
7. *We may convert unlike decimals into like decimals by annexing the requisite number of zeros at the end of the decimal part.*
8. *Comparing Decimals:*
   Step 1. *Convert the given decimals into like decimals.*
   Step 2. *First compare the whole-number parts. The decimal having larger whole-number part is larger than the other.*
   Step 3. *If the whole-number parts are equal, compare the tenths digits. The decimal having bigger digit in the tenths place is the larger one.*
   If the tenths digits are equal, compare the hundredths digits, and so on.
9. *Addition of Decimals:*
   Step 1. *Convert the given decimals into like decimals.*
   Step 2. *Write the addends one under the other so that the decimal points of all the addends are in the same column.*

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Step 3. *Add as in case of whole numbers.*

Step 4. *In the sum, put the decimal point directly under the decimal points in the addends.*

10. *Subtraction of Decimals:*

Step 1. *Convert the given decimals into like decimals.*

Step 2. *Write the smaller number under the larger one so that their decimal points are in the same column.*

Step 3. *Subtract as in the case of whole numbers.*

Step 4. *In the difference, put the decimal point directly under the decimal points of the given number.*

11. *Multiplication of Decimals by 10, 100, 1000, etc.*

Rules: (i) *On multiplying a decimal by 10, the decimal point is shifted to the right by one place.*

(ii) *On multiplying a decimal by 100, the decimal point is shifted to the right by two places, and so on.*

12. *Multiplication of a Decimal by a Whole Number:*

Step 1. *Multiply the decimal without the decimal point by the given whole number.*

Step 2. *Mark the decimal point in the product to have as many places of decimal as there are in the given decimal.*

13. *Multiplication of a Decimal by a Decimal:*

Step 1. *Multiply the two decimals without the decimal point just like whole numbers.*

Step 2. *Mark the decimal point in the product in such a way that the number of decimal places in the product is equal to the sum of the decimal places in the given decimals.*

14. *Dividing a Decimal by 10, 100, 1000, etc.*

Rules: (i) *On dividing a decimal by 10, the decimal point is shifted to the left by one place.*

(ii) *On dividing a decimal by 100, the decimal point is shifted to the left by two places, and so on.*

15. *Dividing a Decimal by a Whole Number:*

Step 1. *Perform the division by considering the dividend a whole number.*

Step 2. *When the division of whole-number part of the dividend is complete, put the decimal point in the quotient and proceed with the division as in case of whole numbers.*

16. *Dividing a Decimal by a Decimal:*

Step 1. *Convert the divisor into a whole number by multiplying the dividend and the divisor by a suitable power of 10.*

Step 2. *Divide the new dividend by the whole number obtained above.*

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TEST PAPER-3

A. **1.** If the cost of a pen is ₹ 32.50, find the cost of 24 such pens.

2. A bus can cover 64.5 km in an hour. How much distance can it cover in 18 hours?

3. Find the product $0.68 \times 6.5 \times 0.04$.

4. Each bag of cement weighs 48.5 kg. How many such bags will weigh 2231 kg?

5. Divide:

(i) 0.196 by 1.4 (ii) 39.168 by 1.2 (iii) 0.228 by 0.38

6. The product of two decimals is 1.824. If one of them is 0.64, find the other.

7. How many pieces of plywood, each 0.45 cm thick, are required to make a pile 2.43 m high?

8. Each side of a polygon is 3.8 cm in length and its perimeter is 22.8 cm. How many sides does the polygon have?

B. ***Mark (✓) against the correct answer in each of the following:***

9. $2\frac{1}{25} = ?$

(a) 2.4 (b) 2.04 (c) 2.004 (d) none of these

10. $1.008 = ?$

(a) $1\frac{2}{25}$ (b) $1\frac{1}{125}$ (c) $1\frac{2}{125}$ (d) none of these

11. 2 kg 5 g = ?

(a) 2.5 kg (b) 2.05 kg (c) 2.005 kg (d) none of these

12. $.012 \div .15 = ?$

(a) 0.8 (b) 0.08 (c) 0.008 (d) none of these

13. $1.1 \times .1 \times .01 = ?$

(a) .11 (b) .011 (c) .0011 (d) none of these

14. $4.669 \div 2.3 = ?$

(a) 2.3 (b) 2.03 (c) 2.003 (d) none of these

15. What should be added to 2.06 to get 3.1?

(a) 1.4 (b) 1.24 (c) 1.04 (d) none of these

16. What should be subtracted from .1 to get .04?

(a) 0.6 (b) 0.06 (c) 0.006 (d) none of these

C. **17.** ***Fill in the blanks.***

(i) $1.001 \div 14 =$ ...... (ii) $204 \div 0.17 =$ ......

(iii) $0.47 \times 5.3 =$ ...... (iv) $0.023 \times 0.03 =$ ......

(v) $(0.7)^2 =$ ...... (vi) $(0.05)^3 =$ ......

D. **18.** ***Write 'T' for true and 'F' for false for each of the following:***

(i) $0.5 \times 0.05 = 0.25$ (ii) $0.25 \times 0.8 = 0.2$

(iii) $0.35 \div 0.7 = 0.5$ (iv) $.4 \times .4 \times .4 = 0.64$

(v) 6 cm = 0.06 m

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4 Rational Numbers

We have so far studied about the systems of natural numbers, whole numbers, integers and fractions. We shall recall them here and extend our ideas to rational numbers.

NATURAL NUMBERS *The counting numbers are called natural numbers.*

Thus, 1, 2, 3, 4, 5, 6, ..., etc., are all natural numbers.

WHOLE NUMBERS *All natural numbers together with 0 (zero) are called whole numbers.*

Thus, 0, 1, 2, 3, 4, 5, 6, ..., etc., are all whole numbers.

Clearly, every natural number is a whole number but 0 is a whole number which is not a natural number.

INTEGERS *All natural numbers, 0 and negatives of counting numbers are called integers.*

Thus, ..., −5, −4, −3, −2, −1, 0, 1, 2, 3, 4, 5, ..., etc., are all integers.

1, 2, 3, 4, 5, 6, ..., etc., are all positive integers.

−1, −2, −3, −4, −5, −6, ..., etc., are all negative integers.

Zero is an integer which is neither positive nor negative.

Clearly, a positive integer is the same as a natural number.

FRACTIONS *The numbers of the form $\frac{a}{b}$, where a and b are natural numbers, are called fractions.*

Thus, $\frac{2}{3}, \frac{3}{8}, \frac{11}{5}, \frac{102}{23}$, etc., are all fractions.

RATIONAL NUMBERS *The numbers of the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$, are called rational numbers.*

Examples of rational numbers

1. *Each of the numbers* $\frac{3}{-4}, \frac{-6}{17}, \frac{-8}{-3}, \frac{2}{5}$ *is a rational number.*

2. *Zero is a rational number,* since we can write $0 = \frac{0}{1}$, which is the quotient of two integers with a nonzero denominator.

3. *Every natural number is a rational number.*

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We can write,

$$1 = \frac{1}{1},\ 2 = \frac{2}{1},\ 3 = \frac{3}{1}, \text{ and so on.}$$

In general, if n is a natural number, then we can write it as $\frac{n}{1}$, which is a rational number.

4. *Every integer is a rational number.*

If m is an integer, then we can write it as $\frac{m}{1}$, which is clearly a rational number.

Thus, every integer is a rational number.

5. *Every fraction is a rational number.*

Let $\frac{a}{b}$ be a fraction. Then, a and b are whole numbers and $b \neq 0$.

But, every whole number is an integer.

Thus, $\frac{a}{b}$ is the quotient of two integers such that $b \neq 0$.

$\therefore$ $\frac{a}{b}$ is a rational number.

Hence, every fraction is a rational number.

POSITIVE RATIONAL NUMBERS

A rational number is said to be positive if its numerator and denominator are either both positive or both negative.

EXAMPLE Each of the numbers $\frac{5}{7}, \frac{-13}{-8}, \frac{17}{9}, \frac{-72}{-40}, \frac{36}{63}$ is a positive rational number.

NEGATIVE RATIONAL NUMBERS

A rational number is said to be negative if its numerator and denominator are such that one of them is a positive integer and the other is a negative integer.

EXAMPLE Each of the numbers $\frac{-3}{5}, \frac{3}{-5}, \frac{-18}{7}, \frac{18}{-7}$ is a negative rational number.

THREE IMPORTANT PROPERTIES OF RATIONAL NUMBERS

PROPERTY 1. *If $\frac{p}{q}$ is a rational number and m is a nonzero integer, then $\frac{p}{q} = \frac{p \times m}{q \times m}$.*

Thus, *a rational number remains unchanged, if its numerator and denominator are multiplied by the same nonzero integer.*

For example, we have:

$$\frac{-2}{3} = \frac{(-2)\times 2}{3\times 2} = \frac{(-2)\times 3}{3\times 3} = \frac{(-2)\times 4}{3\times 4} = \frac{(-2)\times 5}{3\times 5} = \ldots$$

$$\Rightarrow \frac{-2}{3} = \frac{-4}{6} = \frac{-6}{9} = \frac{-8}{12} = \frac{-10}{15} = \ldots$$

PROPERTY 2. *If $\frac{p}{q}$ is a rational number and m is a common divisor of p and q, then $\frac{p}{q} = \frac{p \div m}{q \div m}$.*

Thus, *on dividing the numerator and denominator of a rational number by a common divisor, it remains unchanged.*

For example, we have:

$$\frac{32}{36} = \frac{32 \div 4}{36 \div 4} = \frac{8}{9}. \qquad \text{[HCF of 32 and 36 is 4]}$$

$$\frac{-27}{63} = \frac{(-27) \div 9}{63 \div 9} = \frac{-3}{7}. \qquad \text{[HCF of 27 and 63 is 9]}$$

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Equivalent Rational Numbers

On multiplying the numerator and denominator of a given rational number by the same nonzero number, we get a rational number *equivalent* to the given rational number.

Similarly, on dividing the numerator and denominator of a given rational number by a common divisor, we get a rational number *equivalent* to the given rational number.

Thus, *two rational numbers are said to be equivalent if one can be obtained from the other by multiplying (or dividing) its numerator and denominator by the same nonzero number.*

Thus, equivalent rational numbers are equal.

SOLVED EXAMPLES

EXAMPLE 1. *Find four rational numbers equivalent to each of the rational numbers:*

(i) $\frac{3}{4}$ (ii) $\frac{5}{-7}$ (iii) $\frac{-8}{3}$

Solution We have:

(i) $\frac{3}{4}=\frac{3\times2}{4\times2}=\frac{3\times3}{4\times3}=\frac{3\times4}{4\times4}=\frac{3\times5}{4\times5}$.

$\therefore\ \frac{3}{4}=\frac{6}{8}=\frac{9}{12}=\frac{12}{16}=\frac{15}{20}$.

Thus, four rational numbers equivalent to $\frac{3}{4}$ are $\frac{6}{8},\frac{9}{12},\frac{12}{16}$ and $\frac{15}{20}$.

(ii) $\frac{5}{-7}=\frac{5\times2}{(-7)\times2}=\frac{5\times3}{(-7)\times3}=\frac{5\times4}{(-7)\times4}=\frac{5\times5}{(-7)\times5}$.

$\therefore\ \frac{5}{-7}=\frac{10}{-14}=\frac{15}{-21}=\frac{20}{-28}=\frac{25}{-35}$.

Thus, four rational numbers equivalent to $\frac{5}{-7}$ are

$\frac{10}{-14},\frac{15}{-21},\frac{20}{-28}$ and $\frac{25}{-35}$.

(iii) $\frac{-8}{3}=\frac{(-8)\times2}{3\times2}=\frac{(-8)\times3}{3\times3}=\frac{(-8)\times4}{3\times4}=\frac{(-8)\times5}{3\times5}$.

$\therefore\ \frac{-8}{3}=\frac{-16}{6}=\frac{-24}{9}=\frac{-32}{12}=\frac{-40}{15}$.

Thus, four rational numbers equivalent to $\frac{-8}{3}$ are

$\frac{-16}{6},\frac{-24}{9},\frac{-32}{12}$ and $\frac{-40}{15}$.

An Important Result

If the denominator of a rational number is negative then we multiply its numerator and denominator by −1 to get an equivalent rational number with positive denominator.

EXAMPLE 2. *Write each of the following rational numbers with positive denominator:*

$\frac{3}{-8},\frac{7}{-12},\frac{-5}{-2},\frac{-13}{-8}$

Solution We have:

$\frac{3}{-8}=\frac{3\times(-1)}{(-8)\times(-1)}=\frac{-3}{8}$;

$\frac{7}{-12}=\frac{7\times(-1)}{(-12)\times(-1)}=\frac{-7}{12}$;

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$$\frac{-5}{-2}=\frac{(-5)\times(-1)}{(-2)\times(-1)}=\frac{5}{2};$$

$$\frac{-13}{-8}=\frac{(-13)\times(-1)}{(-8)\times(-1)}=\frac{13}{8}.$$

EXAMPLE 3. *Express $\frac{-5}{13}$ as a rational number with positive numerator.*

Solution We have:

$$\frac{-5}{13}=\frac{(-5)\times(-1)}{13\times(-1)}=\frac{5}{-13}.$$

EXAMPLE 4. *Express $\frac{-4}{7}$ as a rational number with*

(i) numerator $=-12$, *(ii) numerator* $=20$.

Solution (i) Numerator of $\frac{-4}{7}$ is -4.

By what number should we multiply (-4) to get (-12)?
Clearly, such number is $(-12)\div(-4)=3$.
So, we multiply its numerator and denominator by 3.

$$\therefore\quad \frac{-4}{7}=\frac{(-4)\times 3}{7\times 3}=\frac{-12}{21}.$$

Hence, $\frac{-4}{7}=\frac{-12}{21}$.

(ii) Numerator of $\frac{-4}{7}$ is -4.

By what number should we multiply (-4) to get 20?
Clearly, such number is $(20)\div(-4)=-5$.

$$\therefore\quad \frac{-4}{7}=\frac{(-4)\times(-5)}{7\times(-5)}=\frac{20}{-35}.$$

Hence, $\frac{-4}{7}=\frac{20}{-35}$.

EXAMPLE 5. *Express $\frac{-3}{8}$ as a rational number with*

(i) denominator $=32$, *(ii) denominator* $=-40$.

Solution (i) Denominator of $\frac{-3}{8}$ is 8.

By what number should we multiply 8 to get 32?
Clearly, such number is $32\div 8=4$.
So, we multiply its numerator and denominator by 4.

$$\therefore\quad \frac{-3}{8}=\frac{(-3)\times 4}{8\times 4}=\frac{-12}{32}.$$

Hence, $\frac{-3}{8}=\frac{-12}{32}$.

(ii) Denominator of $\frac{-3}{8}$ is 8.

By what number should we multiply 8 to get (-40)?
Clearly, such number is $(-40)\div 8=-5$.
So, we multiply its numerator and denominator by (-5).

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$\therefore \quad \frac{-3}{8} = \frac{(-3)\times(-5)}{8\times(-5)} = \frac{15}{-40}$.

Hence, $\frac{-3}{8} = \frac{15}{-40}$.

EXAMPLE 6. *Express $\frac{-36}{48}$ as a rational number with denominator = 4.*

Solution Denominator of $\frac{-36}{48}$ is 48.

By what number should we divide 48 to get 4?
Clearly, such number is $48 \div 4 = 12$.
So, we divide its numerator and denominator by 12.

$\therefore \quad \frac{-36}{48} = \frac{(-36)\div 12}{48 \div 12} = \frac{-3}{4}$.

Hence, $\frac{-36}{48} = \frac{-3}{4}$.

EXAMPLE 7. *Express $\frac{27}{-45}$ as a rational number with denominator = 5.*

Solution Denominator of $\frac{27}{-45}$ is -45.

By what number should we divide (-45) to get 5?
Clearly, such number is $(-45) \div 5 = (-9)$.
So, we divide its numerator and denominator by (-9).

$\therefore \quad \frac{27}{-45} = \frac{27 \div (-9)}{(-45)\div(-9)} = \frac{-3}{5}$.

Hence, $\frac{27}{-45} = \frac{-3}{5}$.

STANDARD FORM OF A RATIONAL NUMBER

A rational number $\frac{p}{q}$ is said to be in standard form, if q is positive, and p and q have no common divisor other than 1.

METHOD *In order to express a given rational number in standard form, we first convert it into a rational number whose denominator is positive and then we divide its numerator and denominator by their HCF.*

EXAMPLE 8. *Express each of the following numbers in standard form:*

(i) $\frac{21}{35}$ (ii) $\frac{-32}{40}$

Solution (i) The given number is $\frac{21}{35}$.

HCF of 21 and 35 is 7.

So, we divide its numerator and denominator by 7.

$\therefore \quad \frac{21}{35} = \frac{21 \div 7}{35 \div 7} = \frac{3}{5}$.

Hence, $\frac{21}{35} = \frac{3}{5}$ (in standard form).

21)35(1
−21
14)21(1
−14
7)14(2
−14
×

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(ii) The given number is $\frac{-32}{40}$.

HCF of 32 and 40 is 8.

So, we divide its numerator and denominator by 8.

$$\therefore \quad \frac{-32}{40} = \frac{(-32) \div 8}{40 \div 8} = \frac{-4}{5} \text{ (in standard form).}$$

32)40(1
-32
8)32(4
-32
×

EXAMPLE 9. *Express each of the following numbers in standard form:*

(i) $\frac{22}{-55}$ (ii) $\frac{-36}{-45}$

Solution (i) The given number is $\frac{22}{-55}$.

Its denominator is negative.

So, we multiply its numerator and denominator by (−1).

$$\therefore \quad \frac{22}{-55} = \frac{22 \times (-1)}{(-55) \times (-1)} = \frac{-22}{55}.$$

The HCF of 22 and 55 is 11.

So, we divide its numerator and denominator by 11.

$$\therefore \quad \frac{-22}{55} = \frac{(-22) \div 11}{55 \div 11} = \frac{-2}{5}.$$

Hence, $\frac{22}{-55} = \frac{-22}{55} = \frac{-2}{5}$ (in standard form).

22)55(2
-44
11)22(2
-22
×

(ii) The given number is $\frac{-36}{-45}$.

Its denominator is negative.

So, we multiply its numerator and denominator by (−1).

$$\therefore \quad \frac{-36}{-45} = \frac{(-36) \times (-1)}{(-45) \times (-1)} = \frac{36}{45}.$$

HCF of 36 and 45 is 9.

So, we divide its numerator and denominator by 9.

$$\therefore \quad \frac{36}{45} = \frac{36 \div 9}{45 \div 9} = \frac{4}{5}.$$

Hence, $\frac{-36}{-45} = \frac{36}{45} = \frac{4}{5}$ (in standard form).

36)45(1
-36
9)36(4
-36
×

EXAMPLE 10. *Express* $\frac{-247}{228}$ *in standard form.*

Solution The given number is $\frac{-247}{228}$.

HCF of 247 and 228 is 19.

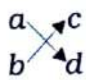
So, we divide its numerator and denominator by 19.

$$\therefore \quad \frac{-247}{228} = \frac{(-247) \div 19}{228 \div 19} = \frac{-13}{12}.$$

Hence, $\frac{-247}{228} = \frac{-13}{12}$ (in standard form).

228)247(1
-228
19)228(12
-19
38
-38
×

PROPERTY 3. *For any two rational numbers* $\frac{a}{b}$ *and* $\frac{c}{d}$, *we have* $\frac{a}{b} = \frac{c}{d} \Leftrightarrow (a \times d) = (b \times c)$.



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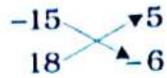
EXAMPLE 11. *Show that $\frac{-15}{18}$ and $\frac{5}{-6}$ are equivalent rational numbers.*

Solution We have:



$(-15)\times(-6) = 90$ and $18\times 5 = 90$

$\Rightarrow (-15)\times(-6) = 18\times 5$

$\Rightarrow \frac{-15}{18} = \frac{5}{-6}.$

Hence, $\frac{-15}{18}$ and $\frac{5}{-6}$ are equivalent.

EXAMPLE 12. *Find x such that $\frac{-3}{8}$ and $\frac{x}{-24}$ are equivalent rational numbers.*

Solution It is given that $\frac{-3}{8} = \frac{x}{-24}$.

$\therefore \frac{-3}{8} = \frac{x}{-24} \Rightarrow 8\times x = (-3)\times(-24)$

$\Rightarrow 8\times x = 72$

$\Rightarrow x = \frac{72}{8} = 9.$

Hence, $x = 9$.

EXERCISE 4A

1. What are rational numbers? Give examples of five positive and five negative rational numbers. Is there any rational number which is neither positive nor negative? Name it.
2. Which of the following are rational numbers?
   (i) $\frac{5}{-8}$ (ii) $\frac{-6}{11}$ (iii) $\frac{7}{15}$ (iv) $\frac{-8}{-12}$ (v) 6
   (vi) -3 (vii) 0 (viii) $\frac{0}{1}$ (ix) $\frac{1}{0}$ (x) $\frac{0}{0}$
3. Write down the numerator and the denominator of each of the following rational numbers:
   (i) $\frac{8}{19}$ (ii) $\frac{5}{-8}$ (iii) $\frac{-13}{15}$ (iv) $\frac{-8}{-11}$ (v) 9
4. Write each of the following integers as a rational number. Write the numerator and the denominator in each case.
   (i) 5 (ii) -3 (iii) 1 (iv) 0 (v) -23
5. Which of the following are positive rational numbers?
   (i) $\frac{3}{-5}$ (ii) $\frac{-11}{15}$ (iii) $\frac{-5}{-8}$ (iv) $\frac{37}{53}$ (v) $\frac{0}{3}$
   (vi) 8
6. Which of the following are negative rational numbers?
   (i) $\frac{-15}{-4}$ (ii) 0 (iii) $\frac{-5}{7}$ (iv) $\frac{4}{-9}$ (v) -6
   (vi) $\frac{1}{-2}$
7. Find four rational numbers equivalent to each of the following.
   (i) $\frac{6}{11}$ (ii) $\frac{-3}{8}$ (iii) $\frac{7}{-15}$ (iv) 8 (v) 1
   (vi) -1

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8. Write each of the following as a rational number with positive denominator.

(i) $\frac{12}{-17}$ (ii) $\frac{1}{-2}$ (iii) $\frac{-8}{-19}$ (iv) $\frac{11}{-6}$

9. Express $\frac{5}{8}$ as a rational number with numerator (i) 15, (ii) –10.

10. Express $\frac{4}{7}$ as a rational number with denominator (i) 21, (ii) –35.

11. Express $\frac{-12}{13}$ as a rational number with numerator (i) –48, (ii) 60.

12. Express $\frac{-8}{11}$ as a rational number with denominator (i) 22, (ii) –55.

13. Express $\frac{14}{-5}$ as a rational number with numerator (i) 56, (ii) –70.

14. Express $\frac{13}{-8}$ as a rational number with denominator (i) –40, (ii) 32.

15. Express $\frac{-36}{24}$ as a rational number with numerator (i) –9, (ii) 6.

16. Express $\frac{84}{-147}$ as a rational number with denominator (i) 7, (ii) –49.

17. Write each of the following rational numbers in standard form:

(i) $\frac{35}{49}$ (ii) $\frac{8}{-36}$ (iii) $\frac{-27}{45}$ (iv) $\frac{-14}{-49}$

(v) $\frac{91}{-78}$ (vi) $\frac{-68}{119}$ (vii) $\frac{-87}{116}$ (viii) $\frac{299}{-161}$

18. Fill in the blanks:

(i) $\frac{-9}{5} = \frac{\dots\dots}{20} = \frac{27}{\dots\dots} = \frac{-45}{\dots\dots}$ (ii) $\frac{-6}{11} = \frac{-18}{\dots\dots} = \frac{\dots\dots}{44}$

19. Which of the following are pairs of equivalent rational numbers?

(i) $\frac{-13}{7}, \frac{39}{-21}$ (ii) $\frac{3}{-8}, \frac{-6}{16}$ (iii) $\frac{9}{4}, \frac{-36}{-16}$

(iv) $\frac{7}{15}, \frac{-28}{60}$ (v) $\frac{3}{12}, \frac{-1}{4}$ (vi) $\frac{2}{3}, \frac{3}{2}$

20. Find x such that:

(i) $\frac{-1}{5} = \frac{8}{x}$ (ii) $\frac{7}{-3} = \frac{x}{6}$ (iii) $\frac{3}{5} = \frac{x}{-25}$

(iv) $\frac{13}{6} = \frac{-65}{x}$ (v) $\frac{16}{x} = -4$ (vi) $\frac{-48}{x} = 2$

21. Which of the following rational numbers are equal?

(i) $\frac{8}{-12}$ and $\frac{-10}{15}$ (ii) $\frac{-3}{9}$ and $\frac{7}{-21}$ (iii) $\frac{-8}{-14}$ and $\frac{15}{21}$

22. State whether the given statement is true or false:

(i) Zero is the smallest rational number.
(ii) Every integer is a rational number.
(iii) The quotient of two integers is always a rational number.
(iv) Every fraction is a rational number.
(v) Every rational number is a fraction.

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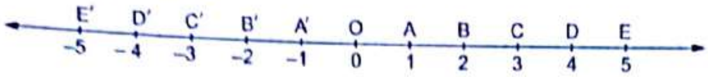
REPRESENTATION OF RATIONAL NUMBERS ON REAL LINE

In the previous class we have learnt how to represent integers on the number line. Let us review it.

Draw any line. Take a point O on it. Call it 0 (zero). Set off equal distances on the right as well as on the left of O. Such a distance is known as a unit length. Clearly, the points A, B, C, D, E represent the integers 1, 2, 3, 4, 5 respectively and the points A', B', C', D', E' represent the integers −1, −2, −3, −4, −5 respectively.



Thus, we may represent any integer by a point on the number line. Clearly, every positive integer lies to the right of O and every negative integer lies to the left of O.

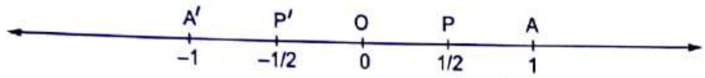
Similarly, we can represent rational numbers.

Consider the following examples.

EXAMPLE 1. *Represent $\frac{1}{2}$ and $-\frac{1}{2}$ on the number line.*

Solution Draw a line. Take a point O on it . Let it represent 0. Set off unit lengths OA and OA' respectively to the right and left of O.

Then, A represents the integer 1 and A' represents the integer −1.



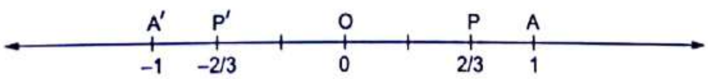
Now, divide OA into two equal parts. Let OP be the first part out of these two parts. Then, the point P represents the rational number $\frac{1}{2}$.

Again, divide OA' into two equal parts. Let OP' be the first part out of these 2 parts. Then, the point P' represents the rational number $-\frac{1}{2}$.

EXAMPLE 2. *Represent $\frac{2}{3}$ and $-\frac{2}{3}$ on the number line.*

Solution Draw a line. Take a point O on it. Let it represent 0. From O set off unit distances OA and OA' to the right and left of O respectively.

Divide OA into 3 equal parts. Let OP be the segment showing 2 parts out of 3. Then, the point P represents the rational number $\frac{2}{3}$.



Again divide OA' into 3 equal parts. Let OP' be the segment consisting of 2 parts out of these 3 parts. Then, the point P' represents the rational number $-\frac{2}{3}$.

EXAMPLE 3. *Represent $\frac{13}{5}$ and $\frac{-13}{5}$ on the number line.*

Solution Draw a line. Take a point O on it. Let it represent 0.

Now, $\frac{13}{5} = 2\frac{3}{5} = 2 + \frac{3}{5}$.

From O, set off unit distances OA, AB and BC to the right of O. Clearly, the points A, B and C represent the integers 1, 2 and 3 respectively. Now, take 2 units OA and AB.

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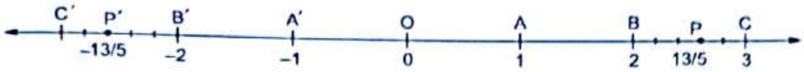
and divide the third unit BC into 5 equal parts. Take 3 parts out of these 5 parts to reach at a point P. Then the point P represents the rational number $\frac{13}{5}$.



Again, from O, set off unit distances to the left. Let these segments be OA', $A'B'$, $B'C'$, etc. Then, clearly the points A', B' and C' represent the integers $-1, -2, -3$ respectively.

Now, $\frac{-13}{5} = -\left(2 + \frac{3}{5}\right)$.

Take 2 full unit lengths to the left of O. Divide the third unit $B'C'$ into 5 equal parts. Take 3 parts out of these 5 parts to reach a point P'.

Then, the point P' represents the rational number $\frac{-13}{5}$.

Thus, we can represent every rational number by a point on the number line.

COMPARISON OF RATIONAL NUMBERS

It is clear that:

(i) every positive rational number is greater than 0,

(ii) every negative rational number is less than 0.

How To Compare Two Rational Numbers?

Step 1. Express each of the two given rational numbers with positive denominator.

Step 2. Take the LCM of these positive denominators.

Step 3. Express each rational number (obtained in Step 1), with this LCM as the common denominator.

Step 4. The number having the greater numerator is greater.

ILLUSTRATIVE EXAMPLES

EXAMPLE 1. *Which of the two rational numbers is greater in each of the following pairs?*

(i) $\frac{2}{5}$ *or 0* *(ii)* $\frac{-3}{8}$ *or 0* *(iii)* $\frac{-9}{5}$ *or 0*

Solution (i) Since every positive rational number is greater than 0, we have: $\frac{2}{5} > 0$.

(ii) Since every negative rational number is less than 0, we have: $\frac{-3}{8} < 0$.

(iii) Since every negative rational number is less than 0, we have: $\frac{-9}{5} < 0$.

EXAMPLE 2. *Which of the two rational numbers* $\frac{-4}{11}$ *and* $\frac{2}{-11}$ *is greater?*

Solution One number $= \frac{-4}{11}$.

The other number $= \frac{2}{-11} = \frac{2 \times (-1)}{(-11) \times (-1)} = \frac{-2}{11}$.

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Since $-4 < -2$, therefore $\frac{-4}{11} < \frac{-2}{11}$.

Hence, $\frac{-4}{11} < \frac{2}{-11}$.

EXAMPLE 3. *Which of the two rational numbers $\frac{2}{-3}$ and $\frac{-4}{5}$ is greater?*

Solution First we write each of the given numbers with a positive denominator.

One number $= \frac{2}{-3} = \frac{2\times(-1)}{(-3)\times(-1)} = \frac{-2}{3}$.

The other number $= \frac{-4}{5}$.

Now, the LCM of the denominators 3 and 5 is 15.

$\therefore \quad \frac{-2}{3} = \frac{(-2)\times 5}{3\times 5} = \frac{-10}{15}$

and $\frac{-4}{5} = \frac{(-4)\times 3}{5\times 3} = \frac{-12}{15}$.

Now, $-10 > -12$

$\Rightarrow \quad \frac{-10}{15} > \frac{-12}{15}$

$\Rightarrow \quad \frac{-2}{3} > \frac{-4}{5}$.

Hence, $\frac{-2}{3} > \frac{-4}{5}$.

EXAMPLE 4. *Arrange the rational numbers $\frac{-3}{5}, \frac{7}{-10}, \frac{-5}{6}$ in ascending order.*

Solution First we express each of the given numbers with positive denominator. We have:

$\frac{7}{-10} = \frac{7\times(-1)}{(-10)\times(-1)} = \frac{-7}{10}$.

So, the given numbers are $\frac{-3}{5}, \frac{-7}{10}, \frac{-5}{6}$.

LCM of 5, 10, 6 $= (2\times 5\times 3) = 30$.

2	5—10—6
5	5—5—3
	1—1—3

Now, $\frac{-3}{5} = \frac{(-3)\times 6}{5\times 6} = \frac{-18}{30}, \frac{-7}{10} = \frac{(-7)\times 3}{10\times 3} = \frac{-21}{30}$

and $\frac{-5}{6} = \frac{(-5)\times 5}{6\times 5} = \frac{-25}{30}$.

Clearly, $\frac{-25}{30} < \frac{-21}{30} < \frac{-18}{30}$, i.e., $\frac{-5}{6} < \frac{-7}{10} < \frac{-3}{5}$.

Hence, $\frac{-5}{6} < \frac{7}{-10} < \frac{-3}{5}$.

EXAMPLE 5. *Arrange the rational numbers $\frac{4}{-9}, \frac{-5}{12}, \frac{7}{-18}$ and $\frac{-2}{3}$ in descending order.*

Solution First we express each of the given numbers with positive denominator. We have:

$\frac{4}{-9} = \frac{4\times(-1)}{(-9)\times(-1)} = \frac{-4}{9}$ and $\frac{7}{-18} = \frac{7\times(-1)}{(-18)\times(-1)} = \frac{-7}{18}$.

So, the given numbers are $\frac{-4}{9}, \frac{-5}{12}, \frac{-7}{18}, \frac{-2}{3}$.

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LCM of 9, 12, 18, 3 = $(3 \times 3 \times 2 \times 2) = 36$.

3	9—12—18—3
3	3—4—6—1
2	1—4—2—1
	1—2—1—1

Now, $\frac{-4}{9} = \frac{(-4) \times 4}{9 \times 4} = \frac{-16}{36}$;

$\frac{-5}{12} = \frac{(-5) \times 3}{12 \times 3} = \frac{-15}{36}$;

$\frac{-7}{18} = \frac{(-7) \times 2}{18 \times 2} = \frac{-14}{36}$;

$\frac{-2}{3} = \frac{(-2) \times 12}{3 \times 12} = \frac{-24}{36}$.

Clearly, $\frac{-14}{36} > \frac{-15}{36} > \frac{-16}{36} > \frac{-24}{36}$

$\Rightarrow \quad \frac{-7}{18} > \frac{-5}{12} > \frac{-4}{9} > \frac{-2}{3}$.

EXAMPLE 6. *List five rational numbers between −2 and −1.*

Solution We may write, $-2 = \frac{-12}{6}$ and $-1 = \frac{-6}{6}$.

Clearly, $\frac{-12}{6} < \frac{-11}{6} < \frac{-10}{6} < \frac{-9}{6} < \frac{-8}{6} < \frac{-7}{6} < \frac{-6}{6}$

$\Rightarrow \quad -2 < \frac{-11}{6} < \frac{-5}{3} < \frac{-3}{2} < \frac{-4}{3} < \frac{-7}{6} < -1$

$\Rightarrow$ five rational numbers between −2 and −1 are

$\frac{-11}{6}, \frac{-5}{3}, \frac{-3}{2}, \frac{-4}{3}$ and $\frac{-7}{6}$.

EXAMPLE 7. *List six rational numbers between −1 and 0.*

Solution We may write, $-1 = \frac{-7}{7}$.

Clearly, $\frac{-7}{7} < \frac{-6}{7} < \frac{-5}{7} < \frac{-4}{7} < \frac{-3}{7} < \frac{-2}{7} < \frac{-1}{7} < 0$

$\Rightarrow \quad -1 < \frac{-6}{7} < \frac{-5}{7} < \frac{-4}{7} < \frac{-3}{7} < \frac{-2}{7} < \frac{-1}{7} < 0$

$\Rightarrow$ six rational numbers between −1 and 0 are

$\frac{-6}{7}, \frac{-5}{7}, \frac{-4}{7}, \frac{-3}{7}, \frac{-2}{7}$ and $\frac{-1}{7}$.

EXAMPLE 8. *List five rational numbers between $\frac{-4}{5}$ and $\frac{-2}{3}$.*

Solution LCM of 5 and 3 is 15.

$\therefore \quad \frac{-4}{5} = \frac{-4}{5} \times \frac{3}{3} = \frac{-12}{15} = \frac{-12 \times 3}{15 \times 3} = \frac{-36}{45}$ and $\frac{-2}{3} = \frac{-2}{3} \times \frac{5}{5} = \frac{-10}{15} = \frac{-10 \times 3}{15 \times 3} = \frac{-30}{45}$.

Clearly, $\frac{-36}{45} < \frac{-35}{45} < \frac{-34}{45} < \frac{-33}{45} < \frac{-32}{45} < \frac{-31}{45} < \frac{-30}{45}$

$\Rightarrow \quad \frac{-4}{5} < \frac{-7}{9} < \frac{-34}{45} < \frac{-11}{15} < \frac{-32}{45} < \frac{-31}{45} < \frac{-2}{3}$

$\Rightarrow$ five rational numbers between $\frac{-4}{5}$ and $\frac{-2}{3}$ are

$\frac{-7}{9}, \frac{-34}{45}, \frac{-11}{15}, \frac{-32}{45}$ and $\frac{-31}{45}$.

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Three Important Properties of Rational Numbers

Property 1. *For each rational number x, exactly one of the following is true:*

(i) $x > 0$ (ii) $x = 0$ (iii) $x < 0$

Property 2. *For any two rational numbers x and y, exactly one of the following is true:*

(i) $x > y$ (ii) $x = y$ (iii) $x < y$

Property 3. *If x, y and z be rational numbers such that $x > y$ and $y > z$ then $x > z$.*

EXERCISE 4B

1. Represent each of the following rational numbers on the number line:

(i) $\frac{1}{3}$ (ii) $\frac{2}{7}$ (iii) $\frac{7}{3}$ (iv) $\frac{22}{7}$ (v) $\frac{37}{8}$

(vi) $\frac{-1}{3}$ (vii) $\frac{-3}{4}$ (viii) $\frac{-12}{7}$ (ix) $\frac{36}{-5}$ (x) $\frac{-43}{9}$

2. Which of the two rational numbers is greater in each of the following pairs?

(i) $\frac{5}{6}$ or 0 (ii) $\frac{-3}{5}$ or 0 (iii) $\frac{5}{8}$ or $\frac{3}{8}$

(iv) $\frac{7}{9}$ or $\frac{-5}{9}$ (v) $\frac{-6}{11}$ or $\frac{5}{-11}$ (vi) $\frac{-15}{4}$ or $\frac{-17}{4}$

3. Which of the two rational numbers is greater in each of the following pairs?

(i) $\frac{5}{9}$ or $\frac{-3}{-8}$ (ii) $\frac{4}{-3}$ or $\frac{-8}{7}$ (iii) $\frac{-12}{5}$ or -3

(iv) $\frac{7}{-9}$ or $\frac{-5}{8}$ (v) $\frac{4}{-5}$ or $\frac{-7}{8}$ (vi) $\frac{9}{-13}$ or $\frac{7}{-12}$

4. Fill in the blanks with the correct symbol out of >, = and <:

(i) $\frac{-3}{7}$ ...... $\frac{6}{-13}$ (ii) $\frac{5}{-13}$ ...... $\frac{-35}{91}$ (iii) -2 ...... $\frac{-13}{5}$

(iv) $\frac{-2}{3}$ ...... $\frac{5}{-8}$ (v) 0 ...... $\frac{-3}{-5}$ (vi) $\frac{-8}{9}$ ...... $\frac{-9}{10}$

5. Arrange the following rational numbers in ascending order:

(i) $\frac{2}{5}, \frac{7}{10}, \frac{8}{15}, \frac{13}{30}$ (ii) $\frac{-3}{4}, \frac{5}{-12}, \frac{-7}{16}, \frac{9}{-24}$

(iii) $\frac{-3}{10}, \frac{7}{-15}, \frac{-11}{20}, \frac{17}{-30}$ (iv) $\frac{2}{3}, \frac{3}{4}, \frac{5}{-6}, \frac{-7}{12}$

6. Arrange the following rational numbers in descending order:

(i) $\frac{-2}{5}, \frac{7}{-10}, \frac{-11}{15}, \frac{19}{-30}$ (ii) $-2, \frac{-13}{6}, \frac{8}{-3}, \frac{1}{3}$

(iii) $\frac{-4}{9}, \frac{5}{-12}, \frac{-7}{18}, \frac{2}{-3}$ (iv) $\frac{17}{-30}, \frac{11}{-15}, \frac{-7}{10}, \frac{3}{5}$

7. Which of the following statements are true?

(i) $\frac{-3}{5}$ lies to the left of 0 on the number line.

(ii) $\frac{-12}{7}$ lies to the right of 0 on the number line.

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(iii) $\frac{1}{3}$ and $\frac{-5}{2}$ lie on opposite sides of 0 on the number line.

(iv) $\frac{-18}{-13}$ lies to the left of 0 on the number line.

(v) $\frac{-5}{-8}$ lies on the right of $\frac{-5}{7}$ on the number line.

8. Find five rational numbers between −3 and −2.
9. Find five rational numbers between −1 and 1.
10. Find five rational numbers between $\frac{-3}{5}$ and $\frac{-1}{2}$.

ADDITION OF RATIONAL NUMBERS

Suppose we have to add two given rational numbers. First convert each of them into a rational number with a positive denominator.

CASE I. When Denominators of Given Numbers are Equal:

Let $\frac{p}{q}$ and $\frac{r}{q}$ be any two rational numbers.

Then, we define $\left(\frac{p}{q}+\frac{r}{q}\right)=\frac{(p+r)}{q}$.

Thus, *in order to add two rational numbers with the same denominator, we simply add their numerators and divide the sum by the common denominator.*

EXAMPLE 1. *Add* $\frac{5}{9}$ *and* $\frac{-13}{9}$.

Solution We have:

$$\frac{5}{9}+\frac{-13}{9}=\frac{5+(-13)}{9}=\frac{-8}{9} \qquad [\because\ 5+(-13)=-8].$$

EXAMPLE 2. *Add* $\frac{7}{-11}$ *and* $\frac{3}{11}$.

Solution We first express $\frac{7}{-11}$ as a rational number with positive denominator. We have:

$$\frac{7}{-11}=\frac{7\times(-1)}{(-11)\times(-1)}=\frac{-7}{11}.$$

$$\therefore\quad \frac{7}{-11}+\frac{3}{11}=\frac{-7}{11}+\frac{3}{11}=\frac{(-7)+3}{11}=\frac{-4}{11} \qquad [\because\ (-7)+3=-4].$$

CASE II. When Denominators of Given Numbers are Unequal:

Step 1. Take the LCM of the denominators of the given rational numbers.

Step 2. Express each of the given rational numbers with the above LCM as the common denominator.

Step 3. Now, add the numbers as shown in Case I.

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EXAMPLE 3. *Add* $\frac{-2}{3}$ *and* $\frac{3}{4}$.

Solution The denominators of the given rational numbers are 3 and 4. LCM of 3 and 4 is 12.

Now, $\frac{-2}{3} = \frac{(-2)\times 4}{3\times 4} = \frac{-8}{12}$ and $\frac{3}{4} = \frac{3\times 3}{4\times 3} = \frac{9}{12}$.

$\therefore \quad \frac{-2}{3} + \frac{3}{4} = \frac{-8}{12} + \frac{9}{12} = \frac{(-8)+9}{12} = \frac{1}{12}$.

EXAMPLE 4. *Simplify:* $\frac{7}{-27} + \frac{11}{18}$.

Solution First we express $\frac{7}{-27}$ as a rational number with positive denominator. We have:

$$\frac{7}{-27} = \frac{7\times(-1)}{(-27)\times(-1)} = \frac{-7}{27}.$$

So, the required sum is $\frac{-7}{27} + \frac{11}{18}$.

LCM of 27 and 18 = $(3\times 3\times 3\times 2) = 54$.

3	27—18
3	9—6
	3—2

$\therefore \quad \frac{-7}{27} = \frac{(-7)\times 2}{27\times 2} = \frac{-14}{54}$ and $\frac{11}{18} = \frac{11\times 3}{18\times 3} = \frac{33}{54}$.

$\therefore \quad \frac{7}{-27} + \frac{11}{18} = \frac{-7}{27} + \frac{11}{18}$

$= \frac{-14}{54} + \frac{33}{54} = \frac{(-14)+33}{54} = \frac{19}{54}$.

Hence, the required sum is $\frac{19}{54}$.

EXAMPLE 5. *Add* $\frac{-3}{8}$ *and* $\frac{-5}{12}$.

Solution The denominators of the given rational numbers are 8 and 12.

LCM of 8 and 12 = $(2\times 2\times 2\times 3) = 24$.

2	8—12
2	4—6
	2—3

Now, $\frac{-3}{8} = \frac{(-3)\times 3}{8\times 3} = \frac{-9}{24}$ and $\frac{-5}{12} = \frac{(-5)\times 2}{12\times 2} = \frac{-10}{24}$.

$\therefore \quad \frac{-3}{8} + \frac{-5}{12} = \frac{-9}{24} + \frac{-10}{24} = \frac{(-9)+(-10)}{24} = \frac{-19}{24}$.

EXAMPLE 6. *Add* $\frac{9}{-16}$ *and* $\frac{-5}{-12}$.

Solution Writing each of the given numbers with positive denominator, we have:

$$\frac{9}{-16} = \frac{9\times(-1)}{(-16)\times(-1)} = \frac{-9}{16} \text{ and } \frac{-5}{-12} = \frac{(-5)\times(-1)}{(-12)\times(-1)} = \frac{5}{12}.$$

So, the required sum is $\frac{-9}{16} + \frac{5}{12}$.

LCM of 16 and 12 = $(2\times 2\times 4\times 3) = 48$.

2	16—12
2	8—6
	4—3

Now, $\frac{-9}{16} = \frac{(-9)\times 3}{16\times 3} = \frac{-27}{48}$ and $\frac{5}{12} = \frac{5\times 4}{12\times 4} = \frac{20}{48}$.

$\therefore \quad \frac{9}{-16} + \frac{-5}{-12} = \frac{-9}{16} + \frac{5}{12} = \frac{-27}{48} + \frac{20}{48} = \frac{(-27)+20}{48} = \frac{-7}{48}$.

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EXAMPLE 7. *Find the sum:* $\left(\frac{-5}{9}+\frac{-7}{12}+\frac{11}{18}\right)$.

Solution The denominators of the given numbers are 9, 12, 18.

LCM of 9, 12, 18 $= (3\times3\times2\times2) = 36$.

3	9—12—18
3	3—4—6
2	1—4—2
	1—2—1

$\therefore \quad \frac{-5}{9}=\frac{(-5)\times4}{9\times4}=\frac{-20}{36}, \frac{-7}{12}=\frac{(-7)\times3}{12\times3}=\frac{-21}{36}$

and $\frac{11}{18}=\frac{11\times2}{18\times2}=\frac{22}{36}$.

$\therefore \quad \frac{-5}{9}+\frac{-7}{12}+\frac{11}{18}=\frac{-20}{36}+\frac{-21}{36}+\frac{22}{36}$

$=\frac{(-20)+(-21)+22}{36}=\frac{(-41)+22}{36}=\frac{-19}{36}$.

Hence, the required sum is $\frac{-19}{36}$.

EXAMPLE 8. *Express each of the following rational numbers as the sum of an integer and a rational number:*

(i) $\frac{19}{6}$ *(ii)* $\frac{-22}{5}$

Solution We have:

(i) $\frac{19}{6}=3\frac{1}{6}=3+\frac{1}{6}$.

(ii) $\frac{-22}{5}=-\left(\frac{22}{5}\right)=-\left(4\frac{2}{5}\right)$

$=-\left(4+\frac{2}{5}\right)=-4+\left(\frac{-2}{5}\right)$.

EXAMPLE 9. *Rahul walks* $\frac{2}{3}$ km *from a place P towards east and then from there* $1\frac{5}{6}$ km *towards west. What is his position now from P?*

Solution Let us denote the distance covered towards east by positive sign. Then, the distance covered towards west would be negative.

Rahul's final distance from $P=\left[\frac{2}{3}+\left(-1\frac{5}{6}\right)\right]$ km

$=\left[\frac{2}{3}+\left(\frac{-11}{6}\right)\right]$ km $=\frac{4+(-11)}{6}$ km

$=\frac{-7}{6}$ km $=-1\frac{1}{6}$ km.

$\therefore$ Rahul is at a distance of $1\frac{1}{6}$ km from P towards west.

EXERCISE 4C

1. Add the following rational numbers:

(i) $\frac{12}{7}$ and $\frac{3}{7}$ (ii) $\frac{-2}{5}$ and $\frac{1}{5}$ (iii) $\frac{3}{-8}$ and $\frac{1}{8}$ (iv) $\frac{-5}{11}$ and $\frac{7}{-11}$

(v) $\frac{9}{-13}$ and $\frac{-11}{-13}$ (vi) $\frac{-2}{9}$ and $\frac{-5}{9}$ (vii) $\frac{-17}{9}$ and $\frac{-11}{9}$ (viii) $\frac{-3}{7}$ and $\frac{5}{-7}$

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2. Add the following rational numbers:

(i) $\frac{-2}{5}$ and $\frac{3}{4}$ (ii) $\frac{-5}{9}$ and $\frac{2}{3}$ (iii) -4 and $\frac{1}{2}$ (iv) $\frac{-7}{27}$ and $\frac{5}{18}$

(v) $\frac{-5}{36}$ and $\frac{-7}{12}$ (vi) $\frac{1}{-9}$ and $\frac{4}{-27}$ (vii) $\frac{-9}{24}$ and $\frac{-1}{18}$ (viii) $\frac{27}{-4}$ and $\frac{-15}{8}$

3. Evaluate:

(i) $\frac{-3}{5}+\frac{7}{5}+\frac{-1}{5}$ (ii) $\frac{-12}{7}+\frac{3}{7}+\frac{-2}{7}$ (iii) $\frac{11}{-12}+\frac{3}{-8}+\frac{1}{4}$

(iv) $\frac{-16}{9}+\frac{-5}{12}+\frac{7}{18}$ (v) $-3+\frac{1}{8}+\frac{-2}{5}$ (vi) $\frac{-13}{8}+\frac{5}{16}+\frac{-1}{4}$

4. Simplify:

(i) $\frac{-8}{15}+\frac{2}{-3}$ (ii) $\frac{-7}{10}+\frac{13}{-15}+\frac{27}{20}$ (iii) $-1+\frac{7}{-9}+\frac{11}{12}$

(iv) $\frac{-11}{39}+\frac{5}{26}+2$ (v) $2+\frac{-1}{2}+\frac{-3}{4}$ (vi) $\frac{-9}{11}+\frac{2}{3}+\frac{-3}{4}$

5. Express each of the following rational numbers as the sum of an integer and a rational number:

(i) $\frac{12}{5}$ (ii) $\frac{-11}{7}$ (iii) $\frac{-25}{9}$ (iv) $\frac{-103}{20}$

SUBTRACTION OF RATIONAL NUMBERS

For any two rational numbers $\frac{a}{b}$ and $\frac{c}{d}$, we define:

$$\left(\frac{a}{b}-\frac{c}{d}\right)=\frac{a}{b}+\left(\frac{-c}{d}\right).$$

We say that the *additive inverse of* $\frac{c}{d}$ *is* $\left(\frac{-c}{d}\right)$.

$$\therefore\ \left(\frac{a}{b}-\frac{c}{d}\right)=\frac{a}{b}+\left(\text{additive inverse of } \frac{c}{d}\right).$$

Also, $\frac{a}{b}-\left(\frac{-c}{d}\right)=\frac{a}{b}+\left\{-\left(\frac{-c}{d}\right)\right\}=\left(\frac{a}{b}+\frac{c}{d}\right)$.

Hence, $\frac{a}{b}-\left(\frac{-c}{d}\right)=\frac{a}{b}+\frac{c}{d}$.

SOLVED EXAMPLES

EXAMPLE 1. *Find the additive inverse of:*

(i) $\frac{5}{9}$ *(ii)* $\frac{-15}{7}$ *(iii)* $\frac{8}{-13}$ *(iv)* $\frac{-12}{-13}$

Solution (i) Additive inverse of $\frac{5}{9}$ is $\frac{-5}{9}$.

(ii) Additive inverse of $\frac{-15}{7}$ is $\frac{15}{7}$.

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(iii) $\frac{8}{-13} = \frac{8 \times (-1)}{(-13) \times (-1)} = \frac{-8}{13}$.

So, its additive inverse is $\frac{8}{13}$.

(iv) $\frac{-12}{-13} = \frac{(-12) \times (-1)}{(-13) \times (-1)} = \frac{12}{13}$.

So, its additive inverse is $\frac{-12}{13}$.

EXAMPLE 2. *Subtract (i) $\frac{3}{4}$ from $\frac{2}{3}$, (ii) $\frac{-5}{7}$ from $\frac{-2}{5}$.*

Solution We have:

(i) $\left(\frac{2}{3} - \frac{3}{4}\right) = \frac{2}{3} + \left(\text{additive inverse of } \frac{3}{4}\right)$

$= \frac{2}{3} + \frac{-3}{4} = \frac{8 + (-9)}{12} = \frac{-1}{12}$.

(ii) $\frac{-2}{5} - \left(\frac{-5}{7}\right) = \frac{-2}{5} + \left(\text{additive inverse of } \frac{-5}{7}\right)$

$= \frac{-2}{5} + \frac{5}{7} = \frac{-14 + 25}{35} = \frac{11}{35}$.

EXAMPLE 3. *The sum of two rational numbers is -5. If one of the numbers is $\frac{-13}{6}$, find the other.*

Solution Let the required number be x. Then,

$\frac{-13}{6} + x = -5 \Rightarrow x = -5 - \left(\frac{-13}{6}\right)$

$= \frac{-5}{1} + \frac{13}{6} \quad \left[\because -\left(\frac{-13}{6}\right) = \frac{13}{6}\right]$

$= \frac{-30 + 13}{6} = \frac{-17}{6}$.

Hence, the required number is $\frac{-17}{6}$.

EXAMPLE 4. *What should be added to $\frac{-7}{8}$ to get $\frac{4}{9}$?*

Solution Let the required number to be added be x. Then,

$\frac{-7}{8} + x = \frac{4}{9} \Rightarrow x = \frac{4}{9} - \left(\frac{-7}{8}\right)$

$= \frac{4}{9} + \frac{7}{8} \quad \left[\because -\left(\frac{-7}{8}\right) = \frac{7}{8}\right]$

$= \frac{(32 + 63)}{72} = \frac{95}{72}$.

Hence, the required number is $\frac{95}{72}$.

EXAMPLE 5. *What should be subtracted from $\frac{-2}{3}$ to get $\frac{5}{6}$?*

Solution Let the required number to be subtracted be x. Then,

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$$\frac{-2}{3} - x = \frac{5}{6} \Rightarrow \frac{-2}{3} = \frac{5}{6} + x$$

$$\Rightarrow x = \frac{-2}{3} - \frac{5}{6} = \frac{-2}{3} + \frac{-5}{6}$$

$$= \frac{(-4)+(-5)}{6} = \frac{-9^3}{6_2} = \frac{-3}{2}.$$

Hence, the required number is $\frac{-3}{2}$.

EXERCISE 4D

1. Find the additive inverse of:
   (i) 5 (ii) −9 (iii) $\frac{3}{14}$ (iv) $\frac{-11}{15}$
   (v) $\frac{15}{-4}$ (vi) $\frac{-18}{-13}$ (vii) 0 (viii) $\frac{1}{-6}$

2. Subtract:
   (i) $\frac{3}{4}$ from $\frac{1}{3}$ (ii) $\frac{-5}{6}$ from $\frac{1}{3}$ (iii) $\frac{-8}{9}$ from $\frac{-3}{5}$ (iv) $\frac{-9}{7}$ from −1
   (v) $\frac{-18}{11}$ from 1 (vi) $\frac{-13}{9}$ from 0 (vii) $\frac{-32}{13}$ from $\frac{-6}{5}$ (viii) −7 from $\frac{-4}{7}$
   (ix) $\frac{5}{9}$ from $\frac{-2}{3}$ (x) 5 from $\frac{-3}{5}$

3. Evaluate:
   (i) $\frac{3}{4} - \frac{4}{5}$ (ii) $-3 - \frac{4}{7}$ (iii) $\frac{7}{24} - \frac{19}{36}$ (iv) $\frac{14}{15} - \frac{13}{20}$
   (v) $\frac{4}{9} - \frac{2}{-3}$ (vi) $\frac{7}{11} - \frac{-4}{-11}$ (vii) $\frac{-5}{14} - \frac{-2}{7}$ (viii) $\frac{-5}{-8} - \frac{-3}{4}$

4. Subtract the sum of $\frac{-36}{11}$ and $\frac{49}{22}$ from the sum of $\frac{33}{8}$ and $\frac{-19}{4}$.

5. The sum of two rational numbers is $\frac{4}{21}$. If one of them is $\frac{5}{7}$, find the other.

6. The sum of two rational numbers is $\frac{-3}{8}$. If one of them is $\frac{3}{16}$, find the other.

7. The sum of two rational numbers is −3. If one of them is $\frac{-15}{7}$, find the other.

8. The sum of two rational numbers is $\frac{-4}{3}$. If one of them is −5, find the other.

9. What should be added to $\frac{-3}{8}$ to get $\frac{5}{12}$?

10. What should be added to $\frac{-12}{5}$ to get 3?

11. What should be added to $\frac{-5}{7}$ to get $\frac{-2}{3}$?

12. What should be added to $\frac{2}{9}$ to get −1?

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13. What should be added to $\left(\frac{-13}{4}+\frac{-3}{8}\right)$ to get 1?

14. What should be subtracted from $\frac{-3}{4}$ to get $\frac{5}{6}$?

15. What should be subtracted from $\frac{-2}{3}$ to get $\frac{-5}{6}$?

16. What should be subtracted from $\frac{-3}{4}$ to get 1?

MULTIPLICATION OF RATIONAL NUMBERS

The product of two rational numbers is defined below.

Product of two rational numbers $= \frac{\textit{Product of their numerators}}{\textit{Product of their denominators}}$.

Thus, for any rational numbers $\frac{a}{b}$ and $\frac{c}{d}$, we have:

$$\left(\frac{a}{b}\times\frac{c}{d}\right)=\frac{(a\times c)}{(b\times d)}.$$

SOLVED EXAMPLES

EXAMPLE 1. *Find the product:*

(i) $\frac{2}{3}\times\frac{5}{7}$ (ii) $\frac{3}{4}\times\left(\frac{-5}{8}\right)$ (iii) $\left(\frac{-4}{5}\right)\times 6$

Solution We have:

(i) $\frac{2}{3}\times\frac{5}{7}=\frac{(2\times 5)}{(3\times 7)}=\frac{10}{21}$.

(ii) $\frac{3}{4}\times\left(\frac{-5}{8}\right)=\frac{3\times(-5)}{4\times 8}=\frac{-15}{32}$.

(iii) $\left(\frac{-4}{5}\right)\times 6=\left(\frac{-4}{5}\right)\times\left(\frac{6}{1}\right)=\frac{(-4)\times 6}{5\times 1}=\frac{-24}{5}$.

EXAMPLE 2. *Simplify:* (i) $\frac{-36}{7}\times\frac{-14}{9}$ (ii) $\frac{-8}{13}\times\frac{39}{-4}$

Solution We have:

(i) $\frac{-36}{7}\times\frac{-14}{9}=\frac{(-36)\times(-14)}{7\times 9}$

$=\frac{\cancel{36}^{4}\times\cancel{14}^{2}}{\cancel{7}_{1}\times\cancel{9}_{1}}$ [$\because$ product of two negative integers is positive]

$=\frac{8}{1}=8$.

(ii) First we write $\frac{39}{-4}$ in standard form.

$$\frac{39}{-4}=\frac{39\times(-1)}{-4\times(-1)}=\frac{-39}{4}.$$

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Now, $\frac{-8}{13} \times \frac{39}{-4} = \frac{-8}{13} \times \frac{-39}{4} = \frac{(-8)\times(-39)}{13\times 4}$

$= \frac{\not{8}^2 \times \not{39}^3}{\not{13}_1 \times \not{4}_1}$ [∵ product of two negative integers is positive]

$= \frac{6}{1} = 6.$

EXAMPLE 3. *Simplify:* (i) $\frac{7}{18} \times (-4)$ (ii) $-36 \times \left(\frac{-5}{9}\right)$

Solution We have:

(i) $\frac{7}{18} \times (-4) = \frac{7}{18} \times \left(\frac{-4}{1}\right) = \frac{7\times(-4)}{18\times 1}$

$= \frac{-(7\times \not{4}^2)}{(\not{18}_9 \times 1)}$ [∵ product of a positive integer and a negative integer is a negative integer]

$= \frac{-14}{9}.$

(ii) $-36 \times \frac{-5}{9} = \frac{(-36)}{1} \times \frac{(-5)}{9} = \frac{(-36)\times(-5)}{1\times 9}$

$= \frac{\not{36}^4 \times 5}{\not{9}_1}$ [∵ product of two negative integers is positive]

$= 20.$

EXAMPLE 4. *Simplify:* (i) $\frac{-5}{9} \times \frac{63}{-100}$ (ii) $\frac{-11}{9} \times \frac{-21}{-44}$

Solution (i) Clearly, $\frac{63}{-100} = \frac{63\times(-1)}{(-100)\times(-1)} = \frac{-63}{100}.$

$\therefore \frac{-5}{9} \times \frac{63}{-100} = \frac{-5}{9} \times \frac{-63}{100}$

$= \frac{(-5)\times(-63)}{9\times 100} = \frac{\not{5}^1 \times \not{63}^7}{\not{9}_1 \times \not{100}_{20}}$ [∵ $(-a)\times(-b) = ab$]

$= \frac{7}{20}.$

Hence, $\frac{-5}{9} \times \frac{63}{-100} = \frac{7}{20}.$

(ii) Clearly, $\frac{-21}{-44} = \frac{(-21)\times(-1)}{(-44)\times(-1)} = \frac{21}{44}.$

$\therefore \frac{-11}{9} \times \frac{-21}{-44} = \frac{-11}{9} \times \frac{21}{44}$

$= \frac{(-11)\times 21}{9\times 44} = \frac{-\not{11}^1 \times \not{21}^7}{\not{9}_3 \times \not{44}_4}$ [∵ $(-a)\times b = -(ab)$]

$= \frac{-7}{12}.$

Hence, $\frac{-11}{9} \times \frac{-21}{-44} = \frac{-7}{12}.$

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EXAMPLE 5. *A car is moving at an average speed of $56\frac{4}{5}$ km/h. How much distance will it cover in $7\frac{1}{2}$ hours?*

Solution Distance covered in 1 hour $= 56\frac{4}{5}$ km $= \frac{284}{5}$ km.

Distance covered in $7\frac{1}{2}$ hours $= \left(\frac{284^{142}}{5_1} \times \frac{15^3}{2_1}\right)$km

$= 426$ km.

Hence, the required distance is 426 km.

EXERCISE 4E

1. Multiply:

(i) $\frac{3}{4}$ by $\frac{5}{7}$ (ii) $\frac{9}{8}$ by $\frac{32}{3}$ (iii) $\frac{7}{6}$ by 24

(iv) $\frac{-2}{3}$ by $\frac{6}{7}$ (v) $\frac{-12}{5}$ by $\frac{10}{-3}$ (vi) $\frac{25}{-9}$ by $\frac{3}{-10}$

(vii) $\frac{-7}{10}$ by $\frac{-40}{21}$ (viii) $\frac{-36}{5}$ by $\frac{20}{-3}$ (ix) $\frac{-13}{15}$ by $\frac{-25}{26}$

2. Simplify:

(i) $\frac{3}{20}\times\frac{4}{5}$ (ii) $\frac{-7}{30}\times\frac{5}{14}$ (iii) $\frac{5}{-18}\times\frac{-9}{20}$

(iv) $\frac{-9}{8}\times\frac{-16}{3}$ (v) $-32\times\frac{-7}{36}$ (vi) $\frac{16}{-21}\times\frac{-14}{5}$

3. Simplify:

(i) $\frac{7}{24}\times -48$ (ii) $\frac{-19}{36}\times 16$ (iii) $\frac{-3}{4}\times\frac{4}{3}$

(iv) $-13\times\frac{17}{26}$ (v) $\frac{-13}{5}\times -10$ (vi) $\frac{-9}{16}\times\frac{-64}{27}$

4. Simplify:

(i) $\left(\frac{13}{8}\times\frac{12}{13}\right)+\left(\frac{-4}{9}\times\frac{3}{-2}\right)$ (ii) $\left(\frac{16}{15}\times\frac{-25}{8}\right)+\left(\frac{-14}{27}\times\frac{6}{7}\right)$

(iii) $\left(\frac{6}{55}\times\frac{-22}{9}\right)-\left(\frac{26}{125}\times\frac{-10}{39}\right)$ (iv) $\left(\frac{-12}{7}\times\frac{-14}{27}\right)-\left(\frac{-8}{45}\times\frac{9}{16}\right)$

5. Find the cost of $3\frac{1}{3}$ metres of cloth at ₹ $40\frac{1}{2}$ per metre.

6. A bus is moving at an average speed of $46\frac{2}{3}$ km/h. How much distance will it cover in $2\frac{2}{5}$ hours?

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RECIPROCAL OR MULTIPLICATIVE INVERSE OF A RATIONAL NUMBER

If the product of two rational numbers is 1 then each one is called the reciprocal of the other.

Thus, *the reciprocal of* $\frac{a}{b}$ *is* $\frac{b}{a}$ and we write, $\left(\frac{a}{b}\right)^{-1} = \frac{b}{a}$.

Clearly, (i) reciprocal of 0 does not exist.

(ii) reciprocal of 1 is 1.

(iii) reciprocal of -1 is -1.

EXAMPLE 1. *Write down the reciprocal of:*

(i) $\frac{13}{7}$ (ii) $\frac{-8}{9}$ (iii) -6

Solution We have:

(i) Reciprocal of $\frac{13}{7}$ is $\frac{7}{13}$.

(ii) Reciprocal of $\frac{-8}{9}$ is $\frac{9}{-8}$.

(iii) Reciprocal of -6 is $\frac{1}{-6}$.

Thus, we get:

(i) $\left(\frac{13}{7}\right)^{-1} = \frac{7}{13}$ (ii) $\left(\frac{-8}{9}\right)^{-1} = \frac{9}{-8} = \frac{-9}{8}$ (iii) $(-6)^{-1} = \frac{1}{-6}$.

DIVISION OF RATIONAL NUMBERS

If $\frac{a}{b}$ *and* $\frac{c}{d}$ *are two rational numbers such that* $\frac{c}{d} \neq 0$, *then we define:*

$$\left(\frac{a}{b} \div \frac{c}{d}\right) = \frac{a}{b} \times \left(\text{reciprocal of } \frac{c}{d}\right) = \left(\frac{a}{b} \times \frac{d}{c}\right).$$

EXAMPLE 1. *Simplify:*

(i) $\frac{7}{15} \div \frac{2}{3}$ (ii) $\frac{-8}{35} \div \frac{2}{7}$ (iii) $\frac{16}{21} \div \frac{-4}{3}$ (iv) $\frac{-9}{20} \div \frac{-3}{10}$

Solution We have:

(i) $\frac{7}{15} \div \frac{2}{3} = \frac{7}{15} \times \frac{3}{2} = \frac{7 \times \cancel{3}^1}{\cancel{15}_5 \times 2} = \frac{7}{10}$.

(ii) $\frac{-8}{35} \div \frac{2}{7} = \frac{-8}{35} \times \frac{7}{2} = \frac{(-8) \times 7}{35 \times 2} = \frac{-(\cancel{8}^4 \times \cancel{7}^1)}{(\cancel{35}_5 \times \cancel{2}_1)} = \frac{-4}{5}$.

(iii) $\frac{16}{21} \div \frac{-4}{3} = \frac{16}{21} \times \frac{3}{-4} = \frac{16}{21} \times \frac{-3}{4}$

$= \frac{16 \times (-3)}{21 \times 4} = \frac{-(\cancel{16}^4 \times \cancel{3}^1)}{(\cancel{21}_7 \times \cancel{4}_1)} = \frac{-4}{7}$.

(iv) $\frac{-9}{20} \div \frac{-3}{10} = \frac{-9}{20} \times \frac{10}{-3} = \frac{-9}{20} \times \frac{-10}{3}$

$= \frac{(-9) \times (-10)}{20 \times 3} = \frac{\cancel{9}^3 \times \cancel{10}^1}{\cancel{20}_2 \times \cancel{3}_1} = \frac{3}{2}$.

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EXAMPLE 2. *Fill in the blanks:* $\frac{27}{16} \div (\ldots) = \frac{-15}{8}$.

Solution Let the required number be $\frac{a}{b}$. Then,

$$\frac{27}{16} \div \frac{a}{b} = \frac{-15}{8} \Rightarrow \frac{27}{16} \times \frac{b}{a} = \frac{-15}{8}$$

$$\Rightarrow \frac{b}{a} = \frac{-15}{8} \div \frac{27}{16} = \frac{-15}{8} \times \frac{16}{27}$$

$$\Rightarrow \frac{b}{a} = \frac{(-15) \times 16}{8 \times 27} = \frac{-(\cancel{15}^5 \times \cancel{16}^2)}{(\cancel{8}_1 \times \cancel{27}_9)} = \frac{-10}{9}$$

$$\Rightarrow \frac{a}{b} = \frac{9}{-10} = \frac{9 \times (-1)}{(-10) \times (-1)} = \frac{-9}{10}.$$

Hence, the required number is $\frac{-9}{10}$.

EXAMPLE 3. *The product of two rational numbers is* $\frac{-8}{9}$. *If one of the numbers is* $\frac{-4}{15}$, *find the other.*

Solution Let the required number be x. Then,

$$x \times \frac{-4}{15} = \frac{-8}{9} \Rightarrow x = \frac{-8}{9} \div \frac{-4}{15}$$

$$\Rightarrow x = \frac{-8}{9} \times \frac{15}{-4} = \frac{-8}{9} \times \frac{-15}{4}$$

$$\Rightarrow x = \frac{(-8) \times (-15)}{9 \times 4} = \frac{\cancel{8}^2 \times \cancel{15}^5}{\cancel{9}_3 \times \cancel{4}_1} = \frac{10}{3}.$$

Hence, the other number is $\frac{10}{3}$.

EXAMPLE 4. *By what number should* $\frac{-33}{8}$ *be divided to get* $\frac{-11}{2}$?

Solution Let the required number be x. Then,

$$\frac{-33}{8} \div x = \frac{-11}{2} \Rightarrow \frac{-33}{8} \times \frac{1}{x} = \frac{-11}{2}$$

$$\Rightarrow \frac{1}{x} = \frac{-11}{2} \div \frac{-33}{8} = \frac{-11}{2} \times \frac{8}{-33}$$

$$\Rightarrow \frac{1}{x} = \frac{-11}{2} \times \frac{-8}{33} = \frac{(-11) \times (-8)}{2 \times 33}$$

$$\Rightarrow \frac{1}{x} = \frac{\cancel{11}^1 \times \cancel{8}^4}{\cancel{2}_1 \times \cancel{33}_3} = \frac{4}{3}$$

$$\Rightarrow x = \frac{3}{4}.$$

Hence, the required number is $\frac{3}{4}$.

EXAMPLE 5. *The cost of 15 pencils is* ₹ $37\frac{1}{2}$. *Find the cost of each pencil.*

Solution Cost of 15 pencils = ₹ $\frac{75}{2}$.

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Cost of 1 pencil $= ₹\left(\frac{75}{2} \div 15\right) = ₹\left(\frac{75}{2} \times \frac{1}{15}\right)$

$= ₹\frac{5}{2} = ₹\,2.50.$

Hence, the cost of each pencil is ₹ 2.50.

EXERCISE 4F

1. Find the multiplicative inverse or reciprocal of each of the following:
   (i) 18 (ii) −16 (iii) $\frac{13}{25}$ (iv) $\frac{-17}{12}$
   (v) $\frac{-6}{19}$ (vi) $\frac{-3}{-5}$ (vii) −1 (viii) 0
2. Simplify:
   (i) $\frac{4}{9} \div \left(\frac{-5}{12}\right)$ (ii) $-8 \div \left(\frac{-5}{16}\right)$ (iii) $\frac{-12}{7} \div (-18)$
   (iv) $\left(\frac{-1}{10}\right) \div \left(\frac{-8}{5}\right)$ (v) $\left(\frac{-16}{35}\right) \div \left(\frac{-15}{14}\right)$ (vi) $\left(\frac{-65}{14}\right) \div \left(\frac{13}{-7}\right)$
3. Fill in the blanks:
   (i) $(\ldots\ldots) \div \left(\frac{-7}{5}\right) = \frac{10}{19}$ (ii) $(\ldots\ldots) \div (-3) = \frac{-4}{15}$
   (iii) $\frac{9}{8} \div (\ldots\ldots) = \frac{-3}{2}$ (iv) $(-12) \div (\ldots\ldots) = \frac{-6}{5}$
4. Divide the sum of $\frac{65}{12}$ and $\frac{8}{3}$ by their difference.
5. By what number should $\frac{-44}{9}$ be divided to get $\frac{-11}{3}$?
6. By what number should $\frac{-8}{15}$ be multiplied to get 24?
7. The product of two rational numbers is 10. If one of the numbers is −8, find the other.
8. The product of two rational numbers is −9. If one of the numbers is −12, find the other.
9. The product of two rational numbers is $\frac{-16}{9}$. If one of the numbers is $\frac{-4}{3}$, find the other.
10. By what rational number should $\frac{-8}{39}$ be multiplied to obtain $\frac{5}{26}$?
11. If 24 pairs of trousers of equal size can be prepared with 54 m of cloth, what length of cloth is required for each pair of trousers?
12. How many pieces, each of length $3\frac{3}{4}$ m, can be cut from a rope of length 30 m?
13. The cost of $2\frac{1}{2}$ metres of cloth is ₹ $78\frac{3}{4}$. Find the cost of cloth per metre.

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EXERCISE 4G

OBJECTIVE QUESTIONS

Mark (✓) against the correct answer in each of the following:

1. $\frac{33}{-55}$ in standard form is
   (a) $\frac{3}{-5}$ (b) $\frac{-3}{5}$ (c) $\frac{-33}{55}$ (d) none of these

2. $\frac{-102}{119}$ in standard form is
   (a) $\frac{-4}{7}$ (b) $\frac{-6}{7}$ (c) $\frac{-6}{17}$ (d) none of these

3. If $\frac{x}{6}=\frac{7}{-3}$, then the value of x is
   (a) -14 (b) 14 (c) 21 (d) -21

4. What should be added to $\frac{-5}{9}$ to get 1?
   (a) $\frac{4}{9}$ (b) $\frac{-4}{9}$ (c) $\frac{14}{9}$ (d) $\frac{-14}{9}$

5. What should be subtracted from $\frac{-3}{4}$ to get $\frac{5}{6}$?
   (a) $\frac{19}{12}$ (b) $\frac{-19}{12}$ (c) $\frac{1}{12}$ (d) $\frac{-1}{12}$

6. Which is smaller out of $\frac{5}{-6}$ and $\frac{-7}{12}$?
   (a) $\frac{5}{-6}$ (b) $\frac{-7}{12}$ (c) cannot be compared

7. Which is larger out of $\frac{2}{-3}$ and $\frac{-4}{5}$?
   (a) $\frac{2}{-3}$ (b) $\frac{-4}{5}$ (c) cannot be compared

8. Reciprocal of -6 is
   (a) 6 (b) $\frac{1}{6}$ (c) $\frac{-1}{6}$ (d) none of these

9. Multiplicative inverse of $\frac{-2}{3}$ is
   (a) $\frac{2}{3}$ (b) $\frac{-3}{2}$ (c) $\frac{3}{2}$ (d) none of these

10. $-2\frac{1}{9}-6=?$
    (a) $-8\frac{1}{9}$ (b) $8\frac{1}{9}$ (c) $4\frac{1}{9}$ (d) $-4\frac{1}{9}$

11. $\frac{-6}{13}-\left(\frac{-7}{15}\right)=?$
    (a) $\frac{-181}{195}$ (b) $\frac{181}{195}$ (c) $\frac{1}{195}$ (d) $\frac{-1}{195}$

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12. $-2\frac{1}{3}+4\frac{3}{5}=?$

(a) $-2\frac{4}{15}$ (b) $2\frac{4}{15}$ (c) $-2\frac{1}{5}$ (d) $2\frac{2}{15}$

13. $\frac{2}{3}-1\frac{5}{7}=?$

(a) $1\frac{1}{21}$ (b) $-1\frac{1}{21}$ (c) $\frac{5}{21}$ (d) $\frac{-5}{21}$

14. Which is greater between $\frac{-4}{9}$ and $\frac{-5}{12}$?

(a) $\frac{-4}{9}$ (b) $\frac{-5}{12}$ (c) both are equal

15. $\frac{-9}{14}+?=-1$

(a) $\frac{5}{14}$ (b) $\frac{-5}{14}$ (c) $\frac{1}{7}$ (d) $\frac{-1}{7}$

16. $\frac{5}{4}-\frac{7}{6}-\frac{-2}{3}=?$

(a) $\frac{3}{4}$ (b) $\frac{-3}{4}$ (c) $\frac{-7}{12}$ (d) $\frac{7}{12}$

17. $1\div\frac{1}{2}=?$

(a) $\frac{1}{2}$ (b) 2 (c) $2\frac{1}{2}$ (d) $1\frac{1}{2}$

18. $\frac{-3}{14}\times ?=\frac{5}{12}$

(a) $\frac{-35}{18}$ (b) $\frac{35}{18}$ (c) $\frac{7}{3}$ (d) $\frac{-7}{3}$

19. $0\div\frac{-7}{5}=?$

(a) not defined (b) $\frac{-5}{7}$ (c) 0 (d) $\frac{5}{7}$

20. $\frac{-3}{8}\div 0=?$

(a) $\frac{-3}{8}$ (b) 0 (c) $\frac{-8}{3}$ (d) not defined

Things to Remember

1. *A number of the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$, is called a rational number.*
2. *Every integer is a rational number and every fraction is a rational number.*

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3. *A rational number* $\frac{p}{q}$ *is said to be in standard form if q is positive and the integers p and q have no common divisor other than 1.*
4. *A rational number* $\frac{p}{q}$ *is positive if p and q are either both positive or both negative.*
5. *A rational number* $\frac{p}{q}$ *is negative if one of p and q is positive and the other is negative.*
6. *If* $\frac{p}{q}$ *is a rational number and m is a nonzero integer then* $\frac{p}{q} = \frac{p \times m}{q \times m}$.
   $\frac{p}{q}$ *and* $\frac{p \times m}{q \times m}$ *are known as* ***equivalent rational numbers.***
7. $\frac{p}{q} = \frac{r}{s}$ *only when* $p \times s = q \times r$.
8. *If* $\frac{p}{q}$ *is a rational number and m is a common divisor of both p and q then* $\frac{p}{q} = \frac{p \div m}{q \div m}$.
9. *If there are two rational numbers with a common denominator then the one with the larger numerator is larger than the other.*
10. *Every positive rational number is greater than 0.*
11. *Every negative rational number is less than 0.*
12. *For each rational number x, exactly one of the following is true:*
    (i) $x > 0$ *(ii)* $x = 0$ *(iii)* $x < 0$
13. *For any two rational numbers x and y, exactly one of the following is true:*
    (i) $x > y$ *(ii)* $x = y$ *(iii)* $x < y$
14. *If x, y, z be rational numbers such that* $x > y$ *and* $y > z$ *then* $x > z$.
15. *For any rational numbers* $\frac{p}{q}$ *and* $\frac{r}{q}$, *we define:*

$$\frac{p}{q} + \frac{r}{q} = \frac{(p+r)}{q}.$$

16. $\frac{p}{q} - \frac{r}{s} = \frac{p}{q} + \left(\text{additive inverse of } \frac{r}{s}\right)$.
17. $\left(\frac{p}{q} \times \frac{r}{s}\right) = \frac{(p \times r)}{(q \times s)}$.
18. *Reciprocal of* $\frac{p}{q}$ *is* $\frac{q}{p}$ *and we write,* $\left(\frac{p}{q}\right)^{-1} = \frac{q}{p}$.
19. $\left(\frac{p}{q} \div \frac{r}{s}\right) = \left(\frac{p}{q} \times \frac{s}{r}\right)$.

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TEST PAPER-4

A. 1. Express each of the following rational numbers in standard form:

(i) $\frac{-209}{247}$ (ii) $\frac{-46}{115}$ (iii) $\frac{84}{-147}$

2. List five rational numbers between –2 and –1.
3. The sum of two rational numbers is –4. If one of them is $\frac{-11}{6}$, find the other.
4. What should be added to $\frac{-7}{8}$ to get $\frac{5}{9}$?
5. A car is moving at an average speed of $56\frac{3}{5}$ km per hour. How much distance will it cover in $7\frac{1}{2}$ hours?
6. By what number should $-4\frac{3}{8}$ be divided to obtain $-3\frac{1}{2}$?
7. How many pieces, each of length $3\frac{3}{4}$ m, can be cut from a rope of length 45 m?
8. Find the cost of $3\frac{1}{3}$ m of cloth at ₹ $121\frac{1}{2}$ per metre.

B. ***Mark (✓) against the correct answer in each of the following:***

9. $\frac{55}{-66}$ in standard form is

(a) $\frac{5}{-6}$ (b) $\frac{-5}{6}$ (c) $\frac{-55}{66}$ (d) none of these

10. What should be subtracted from $\frac{-2}{3}$ to get $\frac{3}{4}$?

(a) $\frac{-17}{12}$ (b) $\frac{17}{12}$ (c) $\frac{-12}{17}$ (d) $\frac{-12}{17}$

11. The product of two numbers is $\frac{-1}{6}$. If one of them is $\frac{-5}{8}$, the other number is

(a) $\frac{-4}{15}$ (b) $\frac{4}{15}$ (c) $\frac{15}{4}$ (d) $\frac{-15}{4}$

12. The multiplicative inverse of $\frac{-3}{4}$ is

(a) $\frac{3}{4}$ (b) $\frac{4}{3}$ (c) $\frac{-4}{3}$ (d) none of these

13. $\frac{-9}{14} + ? = -1$

(a) $\frac{5}{14}$ (b) $\frac{-5}{14}$ (c) $\frac{1}{7}$ (d) $\frac{-1}{7}$

14. $78\frac{3}{4} \div 2\frac{1}{2} = ?$

(a) $31\frac{1}{2}$ (b) $39\frac{3}{8}$ (c) $40\frac{1}{3}$ (d) none of these

15. Which is smaller between $\frac{-5}{6}$ and $\frac{-7}{12}$?

(a) $\frac{-5}{6}$ (b) $\frac{-7}{12}$ (c) cannot be compared

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C. 16. ***Fill in the blanks.***

(i) $(\ldots\ldots) + \left(\frac{-7}{5}\right) = \frac{-2}{3}$

(ii) $\left(\frac{-65}{14}\right) + (\ldots\ldots) = 2\frac{1}{2}$

(iii) $\left(\frac{-3}{8}\right) + (\ldots\ldots) = \frac{5}{12}$

(iv) Multiplicative inverse of $-1\frac{3}{4}$ is ....

D. 17. ***Write 'T' for true and 'F' for false for each of the following:***

(i) $\frac{-15}{-11}$ lies to the left of 0 on the number line.

(ii) $\frac{1}{3}$ and $\frac{-3}{2}$ lie on opposite sides of 0 on the number line.

(iii) $\frac{-8}{13}$ lies to the left of 0 on the number line.

(iv) $\frac{-4}{5} > \frac{-2}{3}$.

(v) $\frac{-3}{5}$ is the largest among $\frac{-3}{5}, \frac{-7}{10}$ and $\frac{-5}{6}$.

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5 Exponents

Recall that we may write, $5 \times 5 \times 5 \times 5$ as 5^4 and read it as 5 raised to the power 4.

In 5^4, we call 5 the ***base*** and 4 the ***exponent*** (or ***index***).

Similarly, $(-6) \times (-6) \times (-6)$ is written as $(-6)^3$.

In $(-6)^3$, the base is (-6) and the exponent is 3.

This notation is called ***exponential form*** or ***power notation.***

Similarly, the product of a rational number multiplied several times by itself can be expressed in the same notation.

Thus, $\left(\frac{2}{3}\right)^4 = \left(\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3}\right)$ and $\left(\frac{-2}{3}\right)^4 = \left(\frac{-2}{3}\right) \times \left(\frac{-2}{3}\right) \times \left(\frac{-2}{3}\right) \times \left(\frac{-2}{3}\right)$, etc.

If $\frac{a}{b}$ is a rational number, then

$$\left(\frac{a}{b}\right)^2 = \frac{a}{b} \times \frac{a}{b} = \frac{(a \times a)}{(b \times b)} = \frac{a^2}{b^2}; \left(\frac{a}{b}\right)^3 = \frac{a}{b} \times \frac{a}{b} \times \frac{a}{b} = \frac{(a \times a \times a)}{(b \times b \times b)} = \frac{a^3}{b^3}, \text{ etc.}$$

In general, we have:

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n} \textit{ for every positive integer } n.$$

EXAMPLE 1. *Evaluate:* (i) $\left(\frac{3}{4}\right)^2$ (ii) $\left(\frac{-2}{3}\right)^3$ (iii) $\left(\frac{-4}{5}\right)^5$

Solution We have:

(i) $\left(\frac{3}{4}\right)^2 = \frac{3^2}{4^2} = \frac{3 \times 3}{4 \times 4} = \frac{9}{16}$.

(ii) $\left(\frac{-2}{3}\right)^3 = \frac{(-2)^3}{3^3} = \frac{(-2) \times (-2) \times (-2)}{3 \times 3 \times 3} = \frac{-8}{27}$.

(iii) $\left(\frac{-4}{5}\right)^5 = \frac{(-4)^5}{5^5} = \frac{(-4) \times (-4) \times (-4) \times (-4) \times (-4)}{5 \times 5 \times 5 \times 5 \times 5} = \frac{-1024}{3125}$.

EXAMPLE 2. *Express each of the following rational numbers in exponential form:*

(i) $\frac{81}{256}$ (ii) $\frac{-32}{243}$ (iii) $\frac{-1}{343}$

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Solution We have:

(i) $81 = 3\times3\times3\times3 = 3^4$ and $256 = 4\times4\times4\times4 = 4^4$.

$\therefore \quad \frac{81}{256} = \frac{3^4}{4^4} = \left(\frac{3}{4}\right)^4.$

(ii) $-32 = (-2)\times(-2)\times(-2)\times(-2)\times(-2) = (-2)^5$ and $243 = 3\times3\times3\times3\times3 = 3^5$.

$\therefore \quad \frac{-32}{243} = \frac{(-2)^5}{3^5} = \left(\frac{-2}{3}\right)^5.$

(iii) $343 = 7\times7\times7 = 7^3$ and $(-1) = (-1)\times(-1)\times(-1) = (-1)^3$.

$\therefore \quad \frac{-1}{343} = \frac{(-1)^3}{7^3} = \left(\frac{-1}{7}\right)^3.$

7	343
7	49
	7

Reciprocal of a Rational Number If $\left(\frac{a}{b}\right)$ is a nonzero rational number, then its reciprocal is $\frac{b}{a}$ and we write, $\left(\frac{a}{b}\right)^{-1} = \frac{b}{a}$.

Now, $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m} \Rightarrow$ reciprocal of $\left(\frac{a}{b}\right)^m = \frac{b^m}{a^m} = \left(\frac{b}{a}\right)^m$.

Hence, *the reciprocal of* $\left(\frac{a}{b}\right)^m$ *is* $\left(\frac{b}{a}\right)^m$.

EXAMPLE 3. *Write the reciprocal of each of the following in exponential form:*

(i) $\left(\frac{2}{5}\right)^6$ *(ii)* $\left(\frac{-3}{7}\right)^{89}$ *(iii)* 3^8 *(iv)* $(-5)^{11}$

Solution We know that the reciprocal of $\left(\frac{a}{b}\right)^m$ is $\left(\frac{b}{a}\right)^m$. Therefore,

(i) reciprocal of $\left(\frac{2}{5}\right)^6$ is $\left(\frac{5}{2}\right)^6$.

(ii) reciprocal of $\left(\frac{-3}{7}\right)^{89}$ is $\left(\frac{-7}{3}\right)^{89}$.

(iii) reciprocal of 3^8 = reciprocal of $\left(\frac{3}{1}\right)^8$, which is $\left(\frac{1}{3}\right)^8$.

(iv) reciprocal of $\left(\frac{-5}{1}\right)^{11}$ is $\left(\frac{-1}{5}\right)^{11}$.

LAWS OF EXPONENTS

The laws of exponents on integers can be extended to rational numbers.

Thus, for any rational number $\frac{a}{b}$ and positive integers m and n, we have:

(i) $\left(\frac{a}{b}\right)^m \times \left(\frac{a}{b}\right)^n = \left(\frac{a}{b}\right)^{m+n}$.

(ii) ***If $m > n$, then*** $\left(\frac{a}{b}\right)^m \div \left(\frac{a}{b}\right)^n = \left(\frac{a}{b}\right)^{m-n}$.

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(iii) ***If $m < n$, then*** $\left(\frac{a}{b}\right)^m \div \left(\frac{a}{b}\right)^n = \frac{\left(\frac{a}{b}\right)^m}{\left(\frac{a}{b}\right)^n} = \frac{1}{\left(\frac{a}{b}\right)^{n-m}}$.

(iv) $\left\{\left(\frac{a}{b}\right)^m\right\}^n = \left(\frac{a}{b}\right)^{mn}$.

(v) $\left(\frac{a}{b}\right)^{-n} = \frac{1}{\left(\frac{a}{b}\right)^n} = \frac{1}{\left(\frac{a^n}{b^n}\right)} = \frac{b^n}{a^n} = \left(\frac{b}{a}\right)^n$.

Thus, $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$.

(vi) $\left(\frac{a}{b}\right)^0 = 1$.

(vii) $\left(\frac{a}{b}\right)^{-1} = \frac{b}{a}$.

EXAMPLE 4. *Simplify and express the result in exponential form:*

(i) $\left(\frac{3}{4}\right)^7 \times \left(\frac{3}{4}\right)^5$ *(ii)* $\left(\frac{-2}{5}\right)^{11} \times \left(\frac{-2}{5}\right)^4$

Solution We have:

(i) $\left(\frac{3}{4}\right)^7 \times \left(\frac{3}{4}\right)^5 = \left(\frac{3}{4}\right)^{(7+5)} = \left(\frac{3}{4}\right)^{12}$. $\left[\because \left(\frac{a}{b}\right)^m \times \left(\frac{a}{b}\right)^n = \left(\frac{a}{b}\right)^{m+n}\right]$

(ii) $\left(\frac{-2}{5}\right)^{11} \times \left(\frac{-2}{5}\right)^4 = \left(\frac{-2}{5}\right)^{(11+4)} = \left(\frac{-2}{5}\right)^{15}$. $\left[\because \left(\frac{a}{b}\right)^m \times \left(\frac{a}{b}\right)^n = \left(\frac{a}{b}\right)^{m+n}\right]$

EXAMPLE 5. *Simplify each of the following and express each as a rational number:*

(i) $\left(\frac{2}{3}\right)^4 \times \left(\frac{2}{3}\right)^2$ *(ii)* $\left(\frac{-3}{4}\right)^3 \times \left(\frac{-3}{4}\right)^2$ *(iii)* $\left(\frac{5}{7}\right)^5 \times \left(\frac{5}{7}\right)^{-3}$ *(iv)* $\left(\frac{-3}{5}\right)^{-3} \times \left(\frac{-3}{5}\right)^2$

Solution We have:

(i) $\left(\frac{2}{3}\right)^4 \times \left(\frac{2}{3}\right)^2 = \left(\frac{2}{3}\right)^{(4+2)}$ $\left[\because \left(\frac{a}{b}\right)^m \times \left(\frac{a}{b}\right)^n = \left(\frac{a}{b}\right)^{m+n}\right]$

$= \left(\frac{2}{3}\right)^6 = \frac{2^6}{3^6}$ $\left[\because \left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}\right]$

$= \frac{64}{729}$.

(ii) $\left(\frac{-3}{4}\right)^3 \times \left(\frac{-3}{4}\right)^2 = \left(\frac{-3}{4}\right)^{3+2}$ $\left[\because \left(\frac{a}{b}\right)^m \times \left(\frac{a}{b}\right)^n = \left(\frac{a}{b}\right)^{m+n}\right]$

$= \left(\frac{-3}{4}\right)^5 = \frac{(-3)^5}{4^5} = \frac{(-3)\times(-3)\times(-3)\times(-3)\times(-3)}{4\times4\times4\times4\times4} = \frac{-243}{1024}$.

(iii) $\left(\frac{5}{7}\right)^5 \times \left(\frac{5}{7}\right)^{-3} = \left(\frac{5}{7}\right)^5 \times \frac{1}{\left(\frac{5}{7}\right)^3} = \frac{(5/7)^5}{(5/7)^3} = \left(\frac{5}{7}\right)^{(5-3)} = \left(\frac{5}{7}\right)^2 = \frac{5^2}{7^2} = \frac{25}{49}$.

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(iv) $\left(\frac{-3}{5}\right)^{-3} \times \left(\frac{-3}{5}\right)^{2} = \frac{1}{\left(\frac{-3}{5}\right)^{3}} \times \left(\frac{-3}{5}\right)^{2} = \frac{1}{\left(\frac{-3}{5}\right)^{(3-2)}} = \frac{1}{\left(\frac{-3}{5}\right)^{1}}$

$= \frac{1}{\left(\frac{-3}{5}\right)} = \frac{5}{-3} = \frac{5 \times (-1)}{(-3) \times (-1)} = \frac{-5}{3}.$

EXAMPLE 6. *Express each of the following as a rational number:*

(i) 4^{-3} *(ii)* $(-3)^{-5}$ *(iii)* $\left(\frac{1}{2}\right)^{-4}$ *(iv)* $\left(\frac{-2}{5}\right)^{-3}$

Solution We have:

(i) $4^{-3} = \left(\frac{4}{1}\right)^{-3} = \left(\frac{1}{4}\right)^{3}$ $\left[\because \left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^{n}\right]$

$= \frac{1^3}{4^3} = \frac{1}{64}.$

(ii) $(-3)^{-5} = \left(\frac{-3}{1}\right)^{-5} = \left(\frac{1}{-3}\right)^{5}$ $\left[\because \left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^{n}\right]$

$= \frac{1^5}{(-3)^5} = \frac{1}{-243} = \frac{1 \times (-1)}{(-243) \times (-1)} = \frac{-1}{243}.$

(iii) $\left(\frac{1}{2}\right)^{-4} = \left(\frac{2}{1}\right)^{4}$ $\left[\because \left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^{n}\right]$

$= \frac{2^4}{1^4} = \frac{16}{1} = 16.$

(iv) $\left(\frac{-2}{5}\right)^{-3} = \left(\frac{5}{-2}\right)^{3}$ $\left[\because \left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^{n}\right]$

$= \frac{5^3}{(-2)^3} = \frac{125}{-8} = \frac{125 \times (-1)}{(-8) \times (-1)} = \frac{-125}{8}.$

EXAMPLE 7. *Simplify and express each of the following as a rational number:*

(i) $\left(\frac{3}{2}\right)^{-3} \times \left(\frac{3}{2}\right)^{-2}$ *(ii)* $\left(\frac{3}{4}\right)^{-2} \times \left(\frac{2}{5}\right)^{-3}$ *(iii)* $\left(\frac{-2}{3}\right)^{-4} \times \left(\frac{-3}{5}\right)^{2}$

Solution We have:

(i) $\left(\frac{3}{2}\right)^{-3} \times \left(\frac{3}{2}\right)^{-2} = \left(\frac{2}{3}\right)^{3} \times \left(\frac{2}{3}\right)^{2}$ $\left[\because \left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^{n}\right]$

$= \left(\frac{2}{3}\right)^{(3+2)} = \left(\frac{2}{3}\right)^{5} = \frac{2^5}{3^5} = \frac{32}{243}.$

(ii) $\left(\frac{3}{4}\right)^{-2} \times \left(\frac{2}{5}\right)^{-3} = \left(\frac{4}{3}\right)^{2} \times \left(\frac{5}{2}\right)^{3}$ $\left[\because \left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^{n}\right]$

$= \frac{4^2}{3^2} \times \frac{5^3}{2^3} = \frac{16^2}{9} \times \frac{125}{8_1} = \frac{2 \times 125}{9 \times 1} = \frac{250}{9}.$

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(iii) $\left(\frac{-2}{3}\right)^{-4} \times \left(\frac{-3}{5}\right)^{2} = \left(\frac{3}{-2}\right)^{4} \times \left(\frac{-3}{5}\right)^{2}$ $\left[\because \left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^{n}\right]$

$= \frac{3^4}{(-2)^4} \times \frac{(-3)^2}{5^2} = \frac{81}{16} \times \frac{9}{25} = \frac{81 \times 9}{16 \times 25} = \frac{729}{400}.$

EXAMPLE 8. *Simplify:* $\left\{\left(\frac{-3}{2}\right)^{2}\right\}^{-3}.$

Solution We have:

$\left\{\left(\frac{-3}{2}\right)^{2}\right\}^{-3} = \left(\frac{-3}{2}\right)^{\{2 \times (-3)\}}$ $\left[\because \left\{\left(\frac{a}{b}\right)^{m}\right\}^{n} = \left(\frac{a}{b}\right)^{mn}\right]$

$= \left(\frac{-3}{2}\right)^{-6} = \left(\frac{2}{-3}\right)^{6}$ $\left[\because \left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^{n}\right]$

$= \frac{2^6}{(-3)^6} = \frac{64}{729}.$

EXAMPLE 9. *Simplify:* $\left[\left\{\left(\frac{-1}{3}\right)^{2}\right\}^{-2}\right]^{-1}.$

Solution We have:

$\left[\left\{\left(\frac{-1}{3}\right)^{2}\right\}^{-2}\right]^{-1} = \left[\left(\frac{-1}{3}\right)^{2 \times (-2)}\right]^{-1} = \left[\left(\frac{-1}{3}\right)^{-4}\right]^{-1}$

$= \left(\frac{-1}{3}\right)^{(-4) \times (-1)} = \left(\frac{-1}{3}\right)^{4} = \frac{(-1)^4}{3^4} = \frac{1}{81}.$

EXAMPLE 10. *Simplify:* $(6^{-1} - 8^{-1})^{-1} + (2^{-1} - 3^{-1})^{-1}.$

Solution We have:

$(6^{-1} - 8^{-1})^{-1} + (2^{-1} - 3^{-1})^{-1} = \left(\frac{1}{6} - \frac{1}{8}\right)^{-1} + \left(\frac{1}{2} - \frac{1}{3}\right)^{-1}$

$= \left(\frac{4-3}{24}\right)^{-1} + \left(\frac{3-2}{6}\right)^{-1}$

$= \left(\frac{1}{24}\right)^{-1} + \left(\frac{1}{6}\right)^{-1}$

$= \left(\frac{24}{1}\right) + \left(\frac{6}{1}\right) = (24 + 6) = 30.$

EXAMPLE 11. *Simplify:* $\left\{6^{-1} + \left(\frac{3}{2}\right)^{-1}\right\}^{-1}.$

Solution We have:

$\left\{6^{-1} + \left(\frac{3}{2}\right)^{-1}\right\}^{-1} = \left(\frac{1}{6} + \frac{2}{3}\right)^{-1} = \left(\frac{1+4}{6}\right)^{-1} = \left(\frac{5}{6}\right)^{-1} = \frac{6}{5}.$

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EXAMPLE 12. *Simplify:* $(2^{-1}+5^{-1})^2\times\left(\frac{-5}{8}\right)^{-1}$.

Solution We have:

$$(2^{-1}+5^{-1})^2\times\left(\frac{-5}{8}\right)^{-1}=\left(\frac{1}{2}+\frac{1}{5}\right)^2\times\left(\frac{8}{-5}\right)=\left(\frac{1}{2}\times\frac{5}{1}\right)^2\times\frac{8}{(-5)}=\left(\frac{5}{2}\right)^2\times\frac{8\times(-1)}{(-5)\times(-1)}$$

$$=\frac{25}{4}\times\frac{(-8)}{5}=\frac{25\times(-8)}{4\times5}=\frac{-(25^5\times8^2)}{(4_1\times5_1)}=\frac{-10}{1}=-10.$$

EXAMPLE 13. *Simplify:* $\left(\frac{1}{2}\right)^{-2}+\left(\frac{1}{3}\right)^{-2}+\left(\frac{1}{4}\right)^{-2}$.

Solution We have:

$$\left(\frac{1}{2}\right)^{-2}+\left(\frac{1}{3}\right)^{-2}+\left(\frac{1}{4}\right)^{-2}=\left(\frac{2}{1}\right)^2+\left(\frac{3}{1}\right)^2+\left(\frac{4}{1}\right)^2\qquad\left[\because\left(\frac{a}{b}\right)^{-n}=\left(\frac{b}{a}\right)^n\right]$$

$$=(2^2+3^2+4^2)=(4+9+16)=29.$$

EXAMPLE 14. *By what number should we multiply* 3^{-9} *so that the product is equal to 3?*

Solution Let the required number be x. Then,

$$3^{-9}\times x=3\ \Rightarrow\ x=\frac{3}{3^{-9}}=(3\times3^9)=(3^1\times3^9)=3^{(1+9)}=3^{10}.$$

Hence, the required number is 3^{10}.

EXAMPLE 15. *By what number should we multiply* $(-8)^{-1}$ *to obtain a product equal to* 10^{-1}*?*

Solution Let the required number be x. Then,

$$(-8)^{-1}\times x=(10)^{-1}\ \Rightarrow\ \frac{1}{(-8)}\times x=\frac{1}{10}$$

$$\Rightarrow\ x=\frac{1}{10}\times(-8)=\frac{-4}{5}.$$

Hence, the required number is $\frac{-4}{5}$.

EXAMPLE 16. *By what number should* $(-15)^{-1}$ *be divided so that the quotient is* $(-5)^{-1}$*?*

Solution Let the required number be x. Then,

$$(-15)^{-1}\div x=(-5)^{-1}\Rightarrow\frac{(-15)^{-1}}{x}=(-5)^{-1}$$

$$\Rightarrow x=\frac{(-15)^{-1}}{(-5)^{-1}}=(-15)^{-1}\div(-5)^{-1}$$

$$\Rightarrow x=\frac{1}{(-15)}\div\frac{1}{(-5)}=\frac{1}{(-15)}\times\frac{(-5)}{1}=\frac{1\times(-5)}{(-15)\times1}=\frac{-5}{-15}=\frac{5}{15}=\frac{1}{3}$$

$$\Rightarrow x=3^{-1}.$$

Hence, the required number is 3^{-1}.

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EXAMPLE 17. *Evaluate:*

(i) 5^0 (ii) $(-6)^0$ (iii) (2^0+3^0)

Solution By definition, we have $a^0=1$ for every integer a. Therefore,

(i) $5^0=1$.

(ii) $(-6)^0=1$.

(iii) $(2^0+3^0)=(1+1)=2$.

EXAMPLE 18. *Simplify:* $\dfrac{10\times5^{n+1}+25\times5^n}{3\times5^{n+2}+10\times5^{n+1}}$.

Solution Given expression $=\dfrac{10\times5^{n+1}+25\times5^n}{3\times5^{n+2}+10\times5^{n+1}}=\dfrac{2\times5\times5^{n+1}+5^2\times5^n}{3\times5^{n+2}+2\times5\times5^{n+1}}$

$=\dfrac{2\times5^{n+2}+5^{n+2}}{3\times5^{n+2}+2\times5^{n+2}}=\dfrac{5^{n+2}(2+1)}{5^{n+2}(3+2)}=\dfrac{3}{5}$.

EXAMPLE 19. *If* $9\times3^n=3^6$, *find the value of n.*

Solution $9\times3^n=3^6 \Rightarrow 3^2\times3^n=3^6 \Rightarrow 3^{(2+n)}=3^6$

$\Rightarrow (2+n)=6 \Rightarrow n=(6-2)=4$.

Hence, $n=4$.

EXAMPLE 20. *If* $25^{(n-1)}+100=5^{(2n-1)}$, *find the value of n.*

Solution $25^{(n-1)}+100=5^{(2n-1)} \Rightarrow (5^2)^{(n-1)}+100=5^{(2n-1)}$

$\Rightarrow 5^{2(n-1)}+100=5^{(2n-1)} \Rightarrow 5^{(2n-1)}-5^{(2n-2)}=100$

$\Rightarrow 5^{(2n-2)}\times(5-1)=100 \Rightarrow 5^{(2n-2)}\times4=100$

$\Rightarrow 5^{(2n-2)}=\dfrac{100}{4}=25=5^2 \Rightarrow (2n-2)=2$

$\Rightarrow 2n=4 \Rightarrow n=2$.

Hence, $n=2$.

EXAMPLE 21. *If* $\dfrac{9^n\times3^2\times3^n-(27)^n}{(3^3)^5\times2^3}=\dfrac{1}{27}$, *find the value of n.*

Solution $\dfrac{9^n\times3^2\times3^n-(27)^n}{(3^3)^5\times2^3}=\dfrac{1}{27}$

$\Rightarrow \dfrac{(3^2)^n\times3^2\times3^n-(3^3)^n}{3^{(3\times5)}\times2^3}=\dfrac{1}{(3^3)} \Rightarrow \dfrac{3^{2n}\times3^2\times3^n-3^{3n}}{3^{15}\times2^3}=3^{-3}$

$\Rightarrow [3^{(2n+n+2)}-3^{3n}]=3^{-3}\times3^{15}\times2^3 \Rightarrow 3^{(3n+2)}-3^{3n}=3^{(-3+15)}\times2^3$

$\Rightarrow 3^{3n}(3^2-1)=3^{12}\times2^3 \Rightarrow 3^{3n}\times8=3^{12}\times8$

$\Rightarrow 3^{3n}=\dfrac{3^{12}\times8}{8}=3^{12} \Rightarrow 3n=12 \Rightarrow n=4$.

Hence, $n=4$.

EXERCISE 5A

1. Write each of the following in power notation:

(i) $\dfrac{5}{7}\times\dfrac{5}{7}\times\dfrac{5}{7}\times\dfrac{5}{7}$

(ii) $\left(\dfrac{-4}{3}\right)\times\left(\dfrac{-4}{3}\right)\times\left(\dfrac{-4}{3}\right)\times\left(\dfrac{-4}{3}\right)\times\left(\dfrac{-4}{3}\right)$

(iii) $\left(\dfrac{-1}{6}\right)\times\left(\dfrac{-1}{6}\right)\times\left(\dfrac{-1}{6}\right)$

(iv) $(-8)\times(-8)\times(-8)\times(-8)\times(-8)$

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2. Express each of the following in power notation:

(i) $\frac{25}{36}$ (ii) $\frac{-27}{64}$ (iii) $\frac{-32}{243}$ (iv) $\frac{-1}{128}$

3. Express each of the following as a rational number:

(i) $\left(\frac{2}{3}\right)^5$ (ii) $\left(\frac{-8}{5}\right)^3$ (iii) $\left(\frac{-13}{11}\right)^2$ (iv) $\left(\frac{1}{6}\right)^3$

(v) $\left(\frac{-1}{2}\right)^5$ (vi) $\left(\frac{-3}{2}\right)^4$ (vii) $\left(\frac{-4}{7}\right)^3$ (viii) $(-1)^9$

4. Express each of the following as a rational number:

(i) $(4)^{-1}$ (ii) $(-6)^{-1}$ (iii) $\left(\frac{1}{3}\right)^{-1}$ (iv) $\left(\frac{-2}{3}\right)^{-1}$

5. Find the reciprocal of each of the following:

(i) $\left(\frac{3}{8}\right)^4$ (ii) $\left(\frac{-5}{6}\right)^{11}$ (iii) 6^7 (iv) $(-4)^3$

6. Find the value of each of the following:

(i) 8^0 (ii) $(-3)^0$ (iii) $4^0 + 5^0$ (iv) $6^0 \times 7^0$

7. Simplify each of the following and express each as a rational number:

(i) $\left(\frac{3}{2}\right)^4 \times \left(\frac{1}{5}\right)^2$ (ii) $\left(\frac{-2}{3}\right)^5 \times \left(\frac{-3}{7}\right)^3$ (iii) $\left(\frac{-1}{2}\right)^5 \times 2^3 \times \left(\frac{3}{4}\right)^2$

(iv) $\left(\frac{2}{3}\right)^2 \times \left(\frac{-3}{5}\right)^3 \times \left(\frac{7}{2}\right)^2$ (v) $\left\{\left(\frac{-3}{4}\right)^3 - \left(\frac{-5}{2}\right)^3\right\} \times 4^2$

8. Simplify and express each as a rational number:

(i) $\left(\frac{4}{9}\right)^6 \times \left(\frac{4}{9}\right)^{-4}$ (ii) $\left(\frac{-7}{8}\right)^{-3} \times \left(\frac{-7}{8}\right)^2$ (iii) $\left(\frac{4}{3}\right)^{-3} \times \left(\frac{4}{3}\right)^{-2}$

9. Express each of the following as a rational number:

(i) 5^{-3} (ii) $(-2)^{-5}$ (iii) $\left(\frac{1}{4}\right)^{-4}$ (iv) $\left(\frac{-3}{4}\right)^{-3}$

(v) $(-3)^{-1} \times \left(\frac{1}{3}\right)^{-1}$ (vi) $\left(\frac{5}{7}\right)^{-1} \times \left(\frac{7}{4}\right)^{-1}$ (vii) $(5^{-1} - 7^{-1})^{-1}$ (viii) $\left\{\left(\frac{4}{3}\right)^{-1} - \left(\frac{1}{4}\right)^{-1}\right\}^{-1}$

(ix) $\left\{\left(\frac{3}{2}\right)^{-1} \div \left(\frac{-2}{5}\right)^{-1}\right\}$ (x) $\left(\frac{23}{25}\right)^0$

10. Simplify:

(i) $\left[\left\{\left(\frac{-1}{4}\right)^2\right\}^{-2}\right]^{-1}$ (ii) $\left\{\left(\frac{-2}{3}\right)^2\right\}^3$ (iii) $\left(\frac{-3}{2}\right)^3 \div \left(\frac{-3}{2}\right)^6$ (iv) $\left(\frac{-2}{3}\right)^7 \div \left(\frac{-2}{3}\right)^4$

11. By what number should $(-5)^{-1}$ be multiplied so that the product is $(8)^{-1}$?

12. By what number should 3^{-3} be multiplied to obtain 4?

13. By what number should $(-30)^{-1}$ be divided to get 6^{-1}?

14. Find x such that $\left(\frac{3}{5}\right)^3 \times \left(\frac{3}{5}\right)^{-6} = \left(\frac{3}{5}\right)^{2x-1}$.

15. Simplify: $\frac{3^5 \times 10^5 \times 25}{5^7 \times 6^5}$.

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16. Simplify: $\dfrac{16\times 2^{n+1}-4\times 2^{n}}{16\times 2^{n+2}-2\times 2^{n+2}}$.

17. Find the value of n when:
(i) $5^{2n}\times 5^{3}=5^{9}$ (ii) $8\times 2^{n+2}=32$ (iii) $6^{2n+1}+36=6^{3}$

18. If $2^{n-7}\times 5^{n-4}=1250$, find the value of n.

Hint: $2^{n-4}\times 2^{-3}\times 5^{n-4}=1250 \Rightarrow \dfrac{2^{n-4}\times 5^{n-4}}{2^{3}}=1250 \Rightarrow (2\times 5)^{n-4}=(10)^{4}$.

EXPRESSING LARGE NUMBERS IN STANDARD FORM

A given number is said to be in standard form if it can be expressed as $k\times 10^{n}$, where k is a real number such that $1\leq k<10$ and n is a positive integer.

EXAMPLE 22. *Express each of the following numbers in standard form:*
(i) 270659 *(ii)* 427500000 *(iii)* 6830000000

Solution Each of the given numbers can be expressed in standard form as shown below:
(i) $270659=2.70659\times 10^{5}$.
(ii) $427500000=4.275\times 10^{8}$.
(iii) $6830000000=6.83\times 10^{9}$.

EXAMPLE 23. *Speed of light in vacuum is* 300000000 m/s. *Express it in standard form.*

Solution Speed of light in vacuum = 300000000 m/s
$=(3\times 10^{8})$ m/s (in standard form).

EXAMPLE 24. *Write each of the following numbers in usual form:*
(i) 6.28×10^{6} *(ii)* 8.235×10^{11} *(iii)* 9.2×10^{3}

Solution We have:
(i) $6.28\times 10^{6}=6280000$.
(ii) $8.235\times 10^{11}=823500000000$.
(iii) $9.2\times 10^{3}=9.2\times 1000=9200$.

NUMBERS IN EXPANDED FORM

Consider the number 8604372.

In the place-value chart, we may express it as under:

TL (10^6)	L (10^5)	T-Th (10^4)	Th (10^3)	H (10^2)	T (10^1)	O (10^0)
8	6	0	4	3	7	2

Thus, in expanded form, we can write it as:

$8604372=8\times 10^{6}+6\times 10^{5}+0\times 10^{4}+4\times 10^{3}+3\times 10^{2}+7\times 10^{1}+2\times 10^{0}$.

EXERCISE 5B

1. Express each of the following numbers in standard form:
(i) 538 (ii) 6428000 (iii) 82934000000 (iv) 940000000000
(v) 23000000

2. Express each of the following numbers in standard form:
(i) Diameter of Earth = 12756000 m.

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(ii) Distance between Earth and Moon = 384000000 m.
(iii) Population of India in March 2001 = 1027000000.
(iv) Number of stars in a galaxy = 100000000000.
(v) The present age of universe = 12000000000 years.

3. Write the following numbers in expanded form:
(i) 684502 (ii) 4007185 (iii) 5807294 (iv) 50074

4. Write the numeral whose expanded form is given below:
(i) $6\times10^4+3\times10^3+0\times10^2+7\times10^1+8\times10^0$
(ii) $9\times10^6+7\times10^5+0\times10^4+3\times10^3+4\times10^2+6\times10^1+2\times10^0$
(iii) $8\times10^5+6\times10^4+4\times10^3+2\times10^2+9\times10^1+6\times10^0$

SQUARE ROOT

The square root of a given number is that number whose square is the given number.

The square root of a number a is denoted by $\sqrt{a}$. In exponential form, $\sqrt{a}$ is written as $a^{\frac{1}{2}}$.

EXAMPLES (i) $2\times2=4 \Rightarrow \sqrt{4}=\sqrt{2\times2}=2$.
(ii) $5\times5=25 \Rightarrow \sqrt{25}=\sqrt{5\times5}=5$.

METHOD

In order to find $\sqrt{a}$, express a as product of primes. Take the product of all primes, choosing one out of every pair.

EXAMPLES *Find* (i) $\sqrt{196}$ *and* (ii) $\sqrt{225}$.

Solution We have
(i) $196=(2\times2)\times(7\times7) \Rightarrow \sqrt{196}=\sqrt{(2\times2)\times(7\times7)}=(2\times7)=14$.
(ii) $225=(3\times3)\times(5\times5) \Rightarrow \sqrt{225}=\sqrt{(3\times3)\times(5\times5)}=(3\times5)=15$.
Show that (i) $\sqrt{64}=8$ (ii) $\sqrt{81}=9$ (iii) $\sqrt{100}=10$ (iv) $\sqrt{121}=11$.

EXERCISE 5C

OBJECTIVE QUESTIONS

Mark tick (✓) against the correct answer in each of the following:

1. $(6^{-1}-8^{-1})^{-1}=?$
(a) $-\frac{1}{2}$ (b) -2 (c) $\frac{1}{24}$ (d) 24

2. $(5^{-1}\times3^{-1})^{-1}=?$
(a) $\frac{1}{15}$ (b) $\frac{-1}{15}$ (c) 15 (d) -15

3. $(2^{-1}-4^{-1})^2=?$
(a) 4 (b) -4 (c) $\frac{1}{16}$ (d) $\frac{-1}{16}$

4. $\left(\frac{1}{2}\right)^{-2}+\left(\frac{1}{3}\right)^{-2}+\left(\frac{1}{4}\right)^{-2}=?$
(a) $\frac{61}{144}$ (b) 29 (c) $\frac{144}{61}$ (d) none of these

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5. $\left\{6^{-1}+\left(\frac{3}{2}\right)^{-1}\right\}^{-1}=?$

(a) $\frac{2}{3}$ (b) $\frac{5}{6}$ (c) $\frac{6}{5}$ (d) none of these

6. $\left(\frac{-1}{2}\right)^{-6}=?$

(a) -64 (b) 64 (c) $\frac{1}{64}$ (d) $\frac{-1}{64}$

7. $\left\{\left(\frac{3}{4}\right)^{-1}-\left(\frac{1}{4}\right)^{-1}\right\}^{-1}=?$

(a) $\frac{3}{8}$ (b) $\frac{-3}{8}$ (c) $\frac{8}{3}$ (d) $\frac{-8}{3}$

8. $\left[\left\{\left(-\frac{1}{2}\right)^{2}\right\}^{-2}\right]^{-1}=?$

(a) $\frac{1}{16}$ (b) 16 (c) $\frac{-1}{16}$ (d) -16

9. $\left(\frac{5}{6}\right)^{0}=?$

(a) $\frac{6}{5}$ (b) 0 (c) 1 (d) none of these

10. $\left(\frac{2}{3}\right)^{-5}=?$

(a) $\frac{32}{243}$ (b) $\frac{243}{32}$ (c) $\frac{-32}{243}$ (d) $\frac{-243}{32}$

11. $\left\{\left(\frac{1}{3}\right)^{2}\right\}^{4}=?$

(a) $\left(\frac{1}{3}\right)^{6}$ (b) $\left(\frac{1}{3}\right)^{8}$ (c) $\left(\frac{1}{3}\right)^{16}$ (d) $\left(\frac{1}{3}\right)^{24}$

12. $\left(\frac{-3}{2}\right)^{-1}=?$

(a) $\frac{2}{3}$ (b) $\frac{-2}{3}$ (c) $\frac{3}{2}$ (d) none of these

13. $(3^2-2^2)\times\left(\frac{2}{3}\right)^{-3}=?$

(a) $\frac{45}{8}$ (b) $\frac{8}{45}$ (c) $\frac{8}{135}$ (d) $\frac{135}{8}$

14. $\left\{\left(\frac{1}{3}\right)^{-3}-\left(\frac{1}{2}\right)^{-3}\right\}\div\left(\frac{1}{4}\right)^{-3}=?$

(a) $\frac{19}{64}$ (b) $\frac{64}{19}$ (c) $\frac{27}{16}$ (d) none of these

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15. $\left(\frac{-1}{5}\right)^3 \div \left(\frac{-1}{5}\right)^8 = ?$

(a) $\left(-\frac{1}{5}\right)^5$ (b) $\left(\frac{-1}{5}\right)^{11}$ (c) $(-5)^5$ (d) $\left(\frac{1}{5}\right)^5$

16. $\left(\frac{-2}{5}\right)^7 \div \left(\frac{-2}{5}\right)^5 = ?$

(a) $\frac{4}{25}$ (b) $\frac{-4}{25}$ (c) $\left(\frac{-2}{5}\right)^{12}$ (d) $\frac{25}{4}$

17. $\left(\frac{-2}{3}\right)^2 = ?$

(a) $\frac{4}{3}$ (b) $\frac{-2}{9}$ (c) $\frac{4}{9}$ (d) $\frac{-4}{9}$

18. $\left(\frac{-1}{2}\right)^3 = ?$

(a) $\frac{-3}{2}$ (b) $\frac{-1}{8}$ (c) $\frac{-1}{6}$ (d) none of these

19. If $\left(\frac{5}{3}\right)^{-5} \times \left(\frac{5}{3}\right)^{11} = \left(\frac{5}{3}\right)^{8x}$, then $x = ?$

(a) $\frac{-1}{2}$ (b) $\frac{-3}{4}$ (c) $\frac{3}{4}$ (d) $\frac{4}{3}$

20. By what number should $(-8)^{-1}$ be multiplied to get 10^{-1}?

(a) $\frac{4}{5}$ (b) $\frac{-5}{4}$ (c) $\frac{-4}{5}$ (d) none of these

21. Which of the following numbers is in standard form?

(a) 21.56×10^5 (b) 215.6×10^4 (c) 2.156×10^6 (d) none of these

Things to Remember

1. *For any rational number $\frac{a}{b}$ and positive integers m and n, we have:*

(i) $\left(\frac{a}{b}\right)^m \times \left(\frac{a}{b}\right)^n = \left(\frac{a}{b}\right)^{m+n}$.

(ii) If $m > n$, then $\left(\frac{a}{b}\right)^m \div \left(\frac{a}{b}\right)^n = \left(\frac{a}{b}\right)^{m-n}$.

(iii) *If $m < n$, then* $\left(\frac{a}{b}\right)^m \div \left(\frac{a}{b}\right)^n = \frac{1}{\left(\frac{a}{b}\right)^{n-m}}$.

(iv) $\left\{\left(\frac{a}{b}\right)^m\right\}^n = \left(\frac{a}{b}\right)^{mn}$.

(v) $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$ *and* $\left(\frac{a}{b}\right)^{-1} = \frac{b}{a}$.

(vi) $\left(\frac{a}{b}\right)^0 = 1$.

2. *A number is said to be in standard form if it can be written as $(k \times 10^n)$, where k is a real number such that $1 \le k < 10$, and n is a positive integer.*

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TEST PAPER-5

A. 1. Write the reciprocal of:

(i) $\left(\frac{2}{3}\right)^4$ (ii) $\left(\frac{-3}{5}\right)^{61}$ (iii) 2^5 (iv) $(-5)^6$

2. By what number should we multiply $(-6)^{-1}$ to obtain a product equal to 9^{-1}?

3. By what number should $(-20)^{-1}$ be divided to obtain $(-10)^{-1}$?

4. (i) Express 2000000 in standard form.

(ii) Express 6.4×10^5 in usual form.

5. Simplify: $\frac{16 \times 2^{n+1} - 8 \times 2^n}{16 \times 2^{n+2} - 4 \times 2^{n+1}}$

6. If $2^{n-7} \times 5^{n-4} = 1250$, find the value of n.

B. ***Mark (✓) against the correct answer in each of the following:***

7. $\left(\frac{3}{4}\right)^0 = ?$

(a) 0 (b) $\frac{4}{3}$ (c) 1 (d) none of these

8. $\left(\frac{-3}{4}\right)^{-3} = ?$

(a) $\frac{27}{64}$ (b) $\frac{64}{27}$ (c) $\frac{-27}{64}$ (d) $\frac{-64}{27}$

9. $\left(\frac{-5}{3}\right)^{-1} = ?$

(a) $\frac{3}{5}$ (b) $\frac{-3}{5}$ (c) $\frac{5}{3}$ (d) $\frac{-5}{3}$

10. $\left\{\left(\frac{1}{3}\right)^{-3} - \left(\frac{1}{2}\right)^{-3}\right\} = ?$

(a) 19 (b) $\frac{1}{19}$ (c) -19 (d) $\frac{-1}{19}$

11. $\left(\frac{-2}{3}\right)^{10} \div \left(\frac{-2}{3}\right)^{8} = ?$

(a) $\frac{4}{9}$ (b) $\frac{-4}{9}$ (c) $\left(\frac{-2}{3}\right)^{18}$ (d) none of these

12. Which of the following numbers is in standard form?

(a) 32.63×10^4 (b) 326.3×10^3 (c) 3.263×10^5 (d) none of these

C. 13. ***Fill in the blanks.***

(i) If $9 \times 3^n = 3^6$, then $n =$ .... (ii) $(8)^0 = ?$

(iii) $\left(\frac{a}{b}\right)^{-16} =$ ...... (iv) $(-2)^{-5} =$ ......

D. 14. ***Write 'T' for true and 'F' for false for each of the following:***

(i) 645 in standard form is 6.45×10^2. (ii) 27000 in standard form is 27×10^3.

(iii) $(3^0 + 4^0 + 5^0) = 12$. (iv) Reciprocal of 5^6 is 6^5.

(v) If $5^{-1} \times x = 8^{-1}$, then $x = \frac{8}{5}$.

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9 Unitary Method

Unitary Method *A method in which the value of unit quantity is first obtained to find the value of any required quantity, is called unitary method.*

In solving problems on unitary method, we come across two types of *variations*.

(i) *Direct Variation* or *Direct Proportion*

(ii) *Inverse Variation* or *Inverse Proportion*

DIRECT VARIATION

Two quantities a and b are said to ***vary directly,*** *if the ratio* $\frac{a}{b}$ *remains constant.*

EXAMPLES (i) *The cost of articles varies directly as the number of articles.*
(More articles, more cost), (Less articles, less cost)

(ii) *The work done varies directly as the number of men at work.*
(More men at work, more work), (Less men at work, less work)

SOLVED EXAMPLES

EXAMPLE 1. *If the cost of 9 toys is ₹ 333, find the cost of 16 such toys.*

Solution Cost of 9 toys = ₹ 333.

Cost of 1 toy = ₹$\left(\frac{333}{9}\right)$ [less toys, less cost]

= ₹ 37.

Cost of 16 toys = ₹ (37×16) [more toys, more cost]

= ₹ 592.

Hence, the cost of 16 toys is ₹ 592.

EXAMPLE 2. *If 25 metres of cloth costs ₹ 1575, how many metres of it can be bought for ₹ 2016?*

Solution For ₹ 1575, cloth bought = 25 m.

For ₹1, cloth bought = $\frac{25}{1575}$ m [less money, less cloth].

For ₹ 2016, cloth bought = $\left(\frac{25}{1575}\times 2016\right)$m [more money, more cloth]

= 32 m.

Hence, the length of cloth bought for ₹ 2016 is 32 m.

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EXAMPLE 3. *If 13 metres of a uniform iron rod weighs 23.4 kg then what will be the weight of 6 metres of the same rod?*

Solution Weight of 13 m of rod = 23.4 kg.

Weight of 1 m of rod = $\frac{23.4}{13}$ kg [less length, less weight].

Weight of 6 m of rod = $\left(\frac{23.4}{13} \times 6\right)$ kg [more length, more weight]

= 10.8 kg.

Hence, the weight of 6 m of rod is 10.8 kg.

EXAMPLE 4. *The length of the shadow of a 3-m-high pole at a certain time of the day is 3.6 m. What is the height of another pole whose shadow at that time is 54 m long?*

Solution If the length of shadow is 3.6 m, height of the pole = 3 m.

If the length of shadow is 1 m, height of the pole = $\frac{3}{3.6}$ m.

If the length of shadow is 54 m, height of the pole = $\left(\frac{3}{3.6} \times 54\right)$ m = 45 m.

Hence, the height of the pole is 45 m.

EXERCISE 9A

1. If 15 oranges cost ₹ 110, what do 39 oranges cost?
2. If 8 kg sugar costs ₹ 260, how much sugar can be bought for ₹ 877.50?
3. The cost of 37 m of silk is ₹ 6290. What length of this silk can be purchased for ₹ 4420?
4. A worker is paid ₹ 1110 for 6 days. If his total wages during a month are ₹ 4625, for how many days did he work?
5. A car can cover a distance of 357 km on 42 litres of petrol. How far can it travel on 12 litres of petrol?
6. Travelling 900 km by rail costs ₹ 2520. What would be the fare for a journey of 360 km when a person travels by the same class?
7. A train covers a distance of 51 km in 45 minutes. How long will it take to cover 221 km?
8. If 22.5 metres of a uniform iron rod weighs 85.5 kg, what will be the length of 22.8 kg of the same rod?
9. If the weight of 6 sheets of a paper is 162 grams, how many sheets of the same quality of paper would weigh 13.5 kg?
10. 1152 bars of soap can be packed in 8 cartons of the same size. How many such cartons will be required to pack 3888 bars?
11. If the thickness of a pile of 16 cardboards is 44 mm, how many cardboards will be there in a pile which is 71.5 cm thick?
12. At a particular time of a day, a 7-m-high flagstaff casts a shadow which is 8.2 m long. What is the height of the building which casts a shadow 20.5 metres in length at the same moment?
13. 15 men can build a 16.25-m-long wall up to a certain height in one day. How many men should be employed to build a wall of the same height but of length 26 metres in one day?
14. In a hospital, the monthly consumption of milk of 60 patients is 1350 litres. How many patients can be accommodated in the hospital if the monthly ration of milk is raised to 1710 litres, assuming that the quota per head remains the same?
15. The extension in an elastic string varies directly as the weight hung on it. If a weight of 150 g produces an extension of 2.8 cm, what weight would produce an extension of 19.6 cm?

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INVERSE VARIATION

Two quantities a and b are said to vary inversely if the product ab remains constant.

EXAMPLE 1. Suppose a car covers a certain distance at a uniform speed. Then we can say for certain that *more is the speed of the car, less is the time taken to cover this distance.*

Thus, *speed varies inversely as the time taken to cover a certain distance.*

EXAMPLE 2. Suppose a given mass of a gas is subjected to pressure. *As the pressure increases, the volume of the gas decreases.*

Thus, *for a given mass of a gas, pressure varies inversely as its volume.*

Rule of Inverse Variation

If a and b vary inversely then the ratio of any two values of a is equal to the inverse ratio of the corresponding values of b.

SOLVED EXAMPLES

EXAMPLE 1. *If 36 men can finish a piece of work in 25 days, how many days will 15 men take to do it?*

Solution 36 men can finish the work in 25 days.

1 man can finish it in (25×36) days [less men, more days].

$\therefore$ 15 men can finish it in $\frac{(25 \times 36)}{15}$ days = 60 days [more men, less days].

Hence, the required number of days = 60.

EXAMPLE 2. *A contractor has a workforce of 420 men who can finish a certain piece of work in 9 months. How many extra men must he employ to complete the job in 7 months?*

Solution To finish the work in 9 months, men required = 420.

To finish the work in 1 month, men required $= (420 \times 9)$

[less months, more men required].

To finish the work in 7 months, men required $= \frac{(420 \times 9)}{7} = 540$

[more months, less men required].

Hence, the number of extra men required $= (540 - 420) = 120$.

EXAMPLE 3. *A garrison of 500 men had provisions for 24 days. However, a reinforcement of 300 men arrived. For how many days will the food last now?*

Solution New strength $= (500 + 300)$ men = 800 men.

For 500 men, there were provisions for 24 days.

For 1 man, there were provisions for (500×24) days [less men, more days].

For 800 men, there were provisions for $\left(\frac{500 \times 24}{800}\right)$ days, i.e., 15 days.

Hence, the food will last now for 15 days.

EXERCISE 9B

1. If 48 men can dig a trench in 14 days, how long will 28 men take to dig a similar trench?
2. 16 men can reap a field in 30 days. How many men must be engaged to reap the same field in 24 days?

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3. 45 cows can graze a field in 13 days. How many cows will graze the same field in 9 days?
4. 16 horses can consume a certain quantity of corn in 25 days. In how many days would the same quantity be consumed by 40 horses?
5. A girl can finish a book in 25 days if she reads 18 pages of it every day. How many days will she take to finish it, if she reads 15 pages every day?
6. Reeta types 40 words per minute and takes 24 minutes to type a certain document. Her friend Geeta has a typing speed of 48 words per minute. In how much time, will she be able to type the same document?
7. A bus covers a certain distance in 3 hours 20 minutes at an average speed of 45 km/h. How long will it take to cover the same distance at a speed of 36 km/h?
8. At the beginning of a month, a factory has enough materials to make 240 tonnes of steel in a month. If 60 more tonnes of steel is to be made that month, how long will the materials last?
9. A contractor employed 210 men to build a house in 60 days. After 12 days, he was joined by 70 more men. In how many days will the remaining work be finished?
   Hint. *210 men can complete the remaining work in 48 days. Find in how many days 280 men can do it.*
10. A military camp has provisions for 630 men to last for 25 days. How many men must be transferred to another camp so that the food lasts for 30 days?
11. A group of 120 men had provisions for 200 days. After 5 days, 30 men died due to an epidemic. How long will the remaining food last?
    Hint. *The remaining food is sufficient for 120 men for 195 days. Find out for how many days, the food will be sufficient for 90 men.*
12. 1200 soldiers in a fort had enough food for 28 days. After 4 days, some soldiers were transferred to another fort and thus the food lasted for an extra 32 days. How many soldiers left the fort?

EXERCISE 9C

OBJECTIVE QUESTIONS

Mark (✓) against the correct answer in each of the following:

1. If 4.5 m of a uniform rod weighs 17.1 kg, what is the weight of 12 m of such a rod?
   (a) 51.2 kg (b) 53 kg (c) 45.6 kg (d) 56 kg
2. In a map, 0.8 cm represents 8.8 km. How much distance will be represented by 80.5 cm?
   (a) 805 km (b) 855.5 km (c) 644 km (d) none of these
3. In a race, Raghu covers 5 km in 20 minutes, how much distance will he cover in 50 minutes?
   (a) 10.5 km (b) 12 km (c) 12.5 km (d) 13.5 km
4. A garrison of 500 men had provisions for 24 days. However, a reinforcement of 300 men arrived. The food will now last for
   (a) 18 days (b) $17\frac{1}{2}$ days (c) 16 days (d) 15 days
5. If $\frac{4}{5}$ of a cistern is filled in 1 minute, how much more time will be required to fill the rest of it?
   (a) 20 seconds (b) 15 seconds (c) 12 seconds (d) 10 seconds

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6. If 21 cows eat as much as 15 buffaloes, how many cows will eat as much as 35 buffaloes?
(a) 49 (b) 56 (c) 45 (d) none of these

7. A tree, 6 m tall, casts a 4-m-long shadow. At the same time a flag pole casts a 50-m-long shadow. How long is the flag pole?
(a) 50 m (b) 75 m (c) $33\frac{1}{3}$ m (d) none of these

8. 8 men can finish a piece of work in 40 days. If 2 more men join them, the work will be completed in
(a) 30 days (b) 32 days (c) 36 days (d) 25 days

9. If 16 men can reap a field in 30 days, in how many days will 20 men reap the same field?
(a) $10\frac{2}{3}$ days (b) 24 days (c) 25 days (d) $37\frac{1}{2}$ days

10. 10 pipes of the same diameter can fill a tank in 24 minutes. If 2 pipes go out of order, how long will the remaining pipe take to fill the tank?
(a) 40 min (b) 45 min (c) 30 min (d) $19\frac{1}{5}$ min

11. 6 dozen eggs are bought for Rs 108. How much will 132 eggs cost?
(a) Rs 204 (b) Rs 264 (c) Rs 184 (d) Rs 198

12. 12 workers take 4 hours to complete a job. How long would it take 15 workers to complete the job?
(a) 2 hrs 40 min (b) 3 hrs 12 min (c) 3 hrs 24 min (d) 3 hrs 30 min

13. A garrison of 500 men had provisions for 27 days. After 3 days, a reinforcement of 300 men arrived. The remaining food will now last for how many days?
(a) 15 days (b) 16 days (c) $17\frac{1}{2}$ days (d) 18 days

14. A rope makes 140 rounds of the circumference of a cylinder, the radius of whose base is 14 cm. How many times can it go round a cylinder with radius 20 cm?
(a) 28 (b) 17 (c) 98 (d) 200

Hint. *Let the required number of rounds be x.*
More radius of the cylinder, less is the number of rounds.
$\therefore$ 20 : 14 :: 140 : *x*. *Find x.*

15. A worker makes a toy every $\frac{2}{3}$ hour. If he works for $7\frac{1}{3}$ hours, then how many toys will he make?
(a) 22 (b) 18 (c) 16 (d) 11

16. 10 men can finish the construction of a wall in 8 days. How many men are added to finish the work in half a day?
(a) 160 (b) 100 (c) 120 (d) 150

Hint. *Let x men can finish the work in $\frac{1}{2}$ day. Then, less days, more men required (indirect).*
$\therefore$ $\frac{1}{2}$: 8 :: 10 : *x*. *Find x* = 160.
Number of men to be added = (160 − 10) = 150.

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TEST PAPER-9

A. 1. If the cost of 8 toys is ₹192, what will be the cost of 14 such toys?

2. A car can cover 270 km in 15 litres of petrol. How many kilometres will it cover in 8 litres of petrol?

3. If 15 envelopes cost ₹11.25, what is the cost of 20 such envelopes?

4. 24 cows can graze a field in 20 days. How many cows can graze it in 15 days?

5. 8 men can finish a piece of work in 15 hours. In how many hours can 20 men finish it?

6. If $\frac{4}{5}$ of a cistern is filled in 1 minute, how much time will be required to fill the empty cistern?

7. A bus covers a certain distance in 3 hours 20 minutes at an average speed of 45 km per hour. How much time will it take to cover the same distance at a speed of 50 km per hour?

8. In a fort, 120 men had provisions for 30 days. For how many days is the food sufficient for 100 men?

B. ***Mark (✓) against the correct answer in each of the following:***

9. In a map, 1 cm represents 8 km. How much distance will be represented by 80.5 cm?
   (a) 640 km (b) 642 km (c) 644 km (d) 648 km

10. If 16 men can reap a field in 30 days, in how many days will 20 men reap the same field?
   (a) 24 days (b) 25 days (c) $10\frac{2}{3}$ days (d) $37\frac{1}{2}$ days

11. If 21 cows eat as much as 15 buffaloes, how many cows will eat as much as 35 buffaloes?
   (a) 45 (b) 49 (c) 56 (d) 54

12. 45 cows can graze a field in 12 days. How many cows will graze the same field in 9 days?
   (a) 60 days (b) $38\frac{3}{4}$ days (c) 54 days (d) none of these

13. 6 dozen eggs are bought for ₹ 108. How much will 108 eggs cost?
   (a) ₹ 171 (b) ₹ 162 (c) ₹ 153 (d) ₹ 180

C. 14. ***Fill in the blanks.***

(i) If 42 men can dig a trench in 14 days, then 1 man can dig it in ...... days.

(ii) If 15 oranges cost ₹ 60, then 12 oranges cost ₹ ....

(iii) If 10 metres of a uniform rod weighs 18 kg, then the weight of 6 metres of the rod is ...... kg.

(iv) If 12 workers finish a piece of work in 4 hours, then 15 workers will finish it in ......hrs ...... min.

D. 15. ***Write 'T' for true and 'F' for false for each of the following:***

(i) If 10 pipes of the same diameter can fill a tank in 24 minutes, then 8 pipes would fill it in 19 min 20 sec.

(ii) If 8 men can finish a piece of work in 40 days, then 10 men can finish it in 32 days.

(iii) A tree 6 m tall casts a shadow of length 4 m. At the same time a flag casts a shadow of length 50 m. The length of the pole is 75 m.

(iv) If a worker takes $\frac{2}{3}$ hour to make a toy, then he will make 12 toys in 8 hours.

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19 Three-Dimensional Shapes

SOLIDS *The objects having definite shape and size are called solids. A solid occupies a fixed amount of space.*

Solids occur in various shapes, such as a cuboid, a cube, a cylinder, a cone, a sphere, etc.

CUBOID *A solid bounded by six rectangular faces is called a cuboid.*

A matchbox, a chalk box, a brick, a tile, a book, an almirah, etc., are all examples of a cuboid.

Various parts of a cuboid are given below.

(i) Faces *A cuboid has 6 rectangular faces.*

The opposite faces of a cuboid are identical.

(ii) Edges *Two adjacent faces of a cuboid meet in a line segment called an edge of the cuboid.*

A cuboid has 12 edges.

(iii) Vertices *Three edges of a cuboid meet at a point called a vertex.*

A cuboid has 8 vertices.

Thus, *a cuboid has 6 rectangular faces, 12 edges and 8 vertices.*

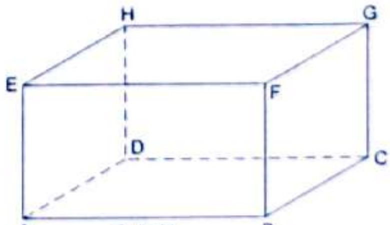
In the given figure, *ABCDEFGH* is a cuboid.

(i) The 6 faces of the cuboid are

ABCD, EFGH, ADHE, BCGF, ABFE, DCGH.

Out of these, four faces, namely, *ABFE, DCGH, BCGF* and *ADHE* are called ***lateral faces*** of the cuboid.

Clearly, a rectangular room is in the form of a cuboid and its 4 walls are its lateral faces.



Cuboid

(ii) The 12 edges of the cuboid are

AB, BC, CD, DA, EF, GH, FG, EH, CG, BF, AE, DH.

(iii) The 8 vertices of the cuboid are *A, B, C, D, E, F, G, H.*

A cuboid has three dimensions, namely, length, breadth and height.

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CUBE

CUBE *A cuboid whose length, breadth and height are all equal is called a cube.* Dice, ice cubes, sugar cubes, etc., are all examples of a cube.

Like cuboid, a cube has 6 faces, 12 edges and 8 vertices.

In the given figure, *ABCDEFGH* is a cube.

(i) 6 faces of the cube are

ABCD, EFGH, ADHE, BCGF, ABFE, DCGH.

Lateral faces are

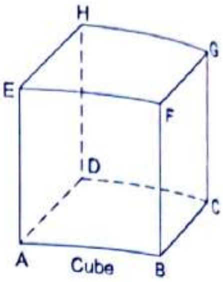
ABFE, DCGH, BCGF, ADHE.

(ii) 12 edges of the cube are

AB, BC, CD, DA, EF, GH, FG, EH, CG, BF, AE, DH.

(iii) 8 vertices of the cube are *A, B, C, D, E, F, G, H.*

Length, breadth and height of a cube are equal.



Cube

CYLINDERS

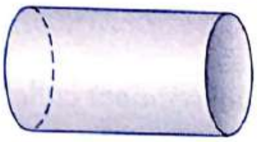
CYLINDER Solids like circular pillars, circular pipes, circular pencils, measuring jars, road rollers and gas cylinders, etc., are said to be in cylindrical shapes.

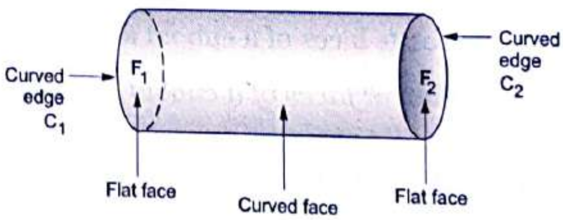
Parts of a Cylinder:

(i) A cylinder has no vertex.

(ii) A cylinder has two curved edges (C_1 and C_2).

(iii) A cylinder has one curved face and two flat faces (F_1 and F_2).


Cylinder



CONE

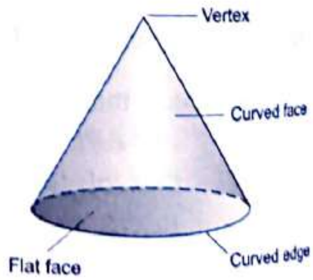
CONE Objects such as ice-cream cone, a conical tent, a conical vessel, a clown's cap, etc., are in the shape of a cone.

Parts of a Cone:

(i) A cone has one vertex.

(ii) A cone has one curved edge.

(iii) A cone has one curved face and one flat face.



SPHERE

SPHERE An object which is in the shape of a ball is said to have the shape of a sphere.

Parts of a Sphere:

(i) A sphere has no vertex.

(ii) A sphere has no edge.

(iii) A sphere has a curved surface.

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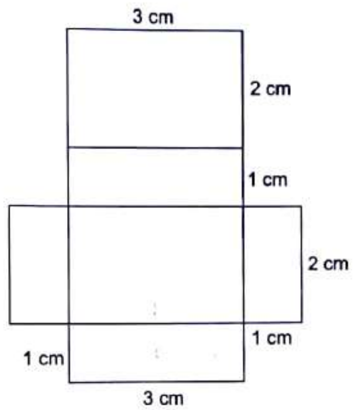
SUMMARY			
Name of the solid	Number of faces	Number of vertices	Number of Edges
(i) Cuboid	6	8	12
(ii) Cube	6	8	12
(iii) Cylinder	3	Nil	2
(iv) Cone	2	1	1
(v) Sphere	1	Nil	Nil

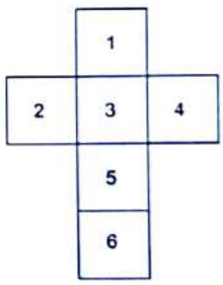
NET OF A THREE-DIMENSIONAL FIGURE

A net of a 3-dimensional figure is the shape that can be cut out of a plane piece of paper or cardboard and folded to make the 3-dimensional shape.

EXAMPLE 1. *Given below is the net of a cuboid of length* = 3 cm, *breadth* = 2 cm *and height* = 1 cm.

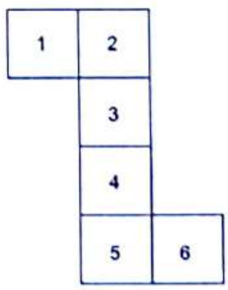


EXAMPLE 2. *The adjoining figure is the net of a cube. Fold these faces to make a cube.*



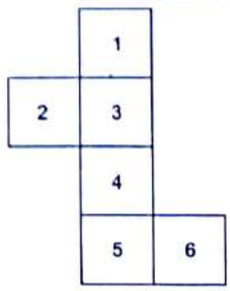
Solution Fold the given faces in such a way that
1 lies opposite 5;
2 lies opposite 4;
3 lies opposite 6.
Thus, a cube is formed.

EXAMPLE 3. *The given figure is the net of a cube. Fold these faces to make a cube.*



Solution Fold the given faces in such a way that
2 lies opposite 4;
3 lies opposite 5;
1 lies opposite 6.
Thus, a cube is formed.

EXAMPLE 4. *The given figure is the net of a cube. Fold these faces to make a cube.*



Solution Fold the given faces in such a way that
1 lies opposite 4;
3 lies opposite 5;
2 lies opposite 6.
Thus, a cube is formed.

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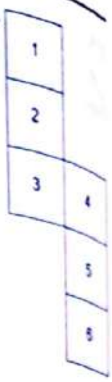
EXAMPLE 5. *The given figure is the net of a cube. Fold these faces to make a cube.*



Solution Fold the given faces in such a way that

1 is the upper face and 3 is the bottom face.

Thus, 1 lies opposite 3;

2 lies opposite 5;

4 lies opposite 6.

Thus, a cube is formed.

EXERCISE 19

1. Fill in the blanks:
   - (i) A cuboid has ...... rectangular faces, ... edges and ...... vertices.
   - (ii) A cylinder has ...... curved face and ...... flat faces.
   - (iii) A cone has one ...... face and one ...... face.
   - (iv) A sphere has a ...... surface.
2. Write (T) for true and (F) for false:
   - (i) A cylinder has no vertex.
   - (ii) A cube has 6 faces, 12 edges and 8 vertices.
   - (iii) A cone has one vertex.
   - (iv) A sphere has one edge.
   - (v) A sphere has a curved surface.
3. Write five examples of each one of (i) a cone (ii) a sphere (iii) a cuboid and (iv) a cylinder.

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